

# 低温物理学講義

家 泰弘

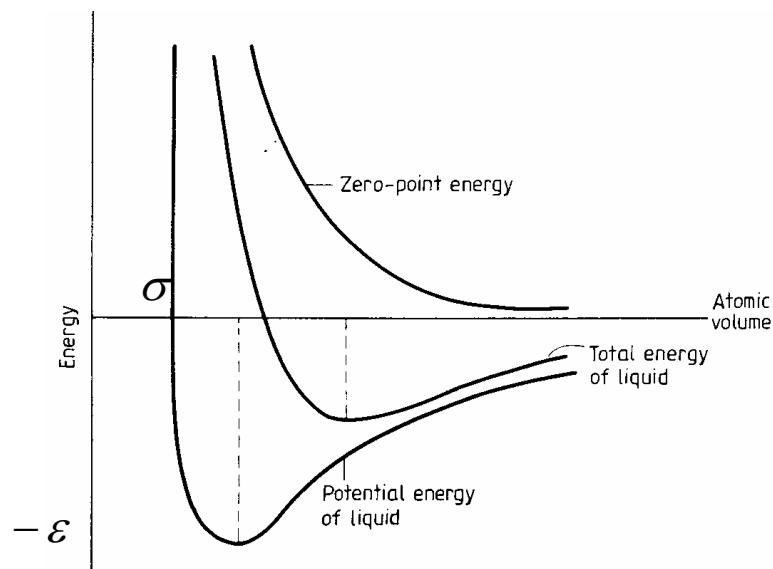
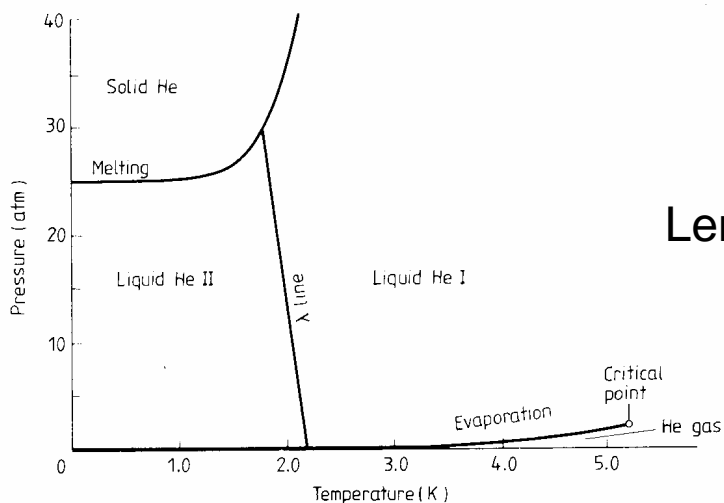
## 超流動 - その1

- ヘリウム, 相図 $^3\text{He}$ と $^4\text{He}$
- 量子パラメーター, ボース凝縮
- 超流動, 粘性, 二流体モデル
- 内部対流, 熱機械効果
- 超流動流体力学
- 第2音波, 第3音波, 第4音波

## 参考書

- D.R.Tilley and J.Tilley, "Superfluidity and Superconductivity" 3rd ed.(Adam Hilger, Bristol and New York, 1990)
- 山田一雄, 大見哲巨「超流動」新物理学シリーズ28 培風館, 1995
- S.J.Putterman: "Superfluid Hydrodynamics" (North Holland, 1974)

# ヘリウム ( $^4\text{He}$ ) の相図



## Lennard-Jones ポテンシャル

$$V(r) = 4\epsilon \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 \right]$$

表 1

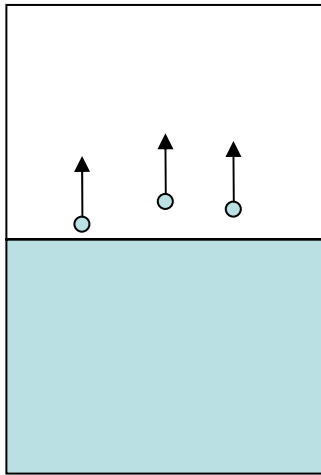
	$\epsilon$ (K)	$\sigma$ (Å)	$\Lambda^*$
$^3\text{He}$	10.2	2.56	3.05
$^4\text{He}$	10.2	2.56	2.64
Ne	35.6	2.74	0.59
Ar	120	3.41	0.19
Kr	166	3.65	0.10
Xe	230	3.92	0.06

絶対零度まで(常圧では)  
液体に留まる唯一の物質  
量子液体

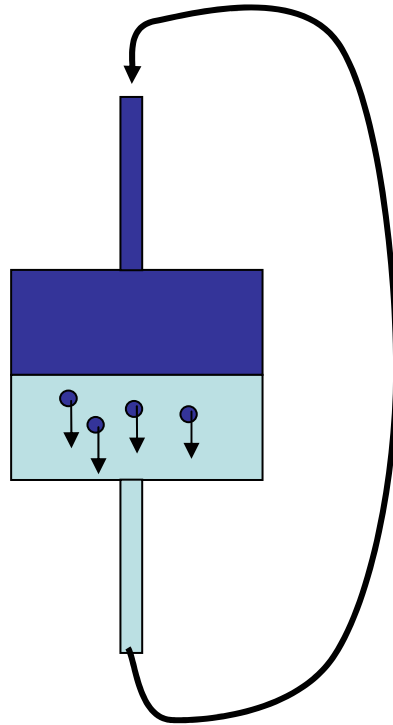
$$\Lambda^* \equiv \frac{\hbar}{\sigma \sqrt{m\epsilon}}$$

# 低温を得る

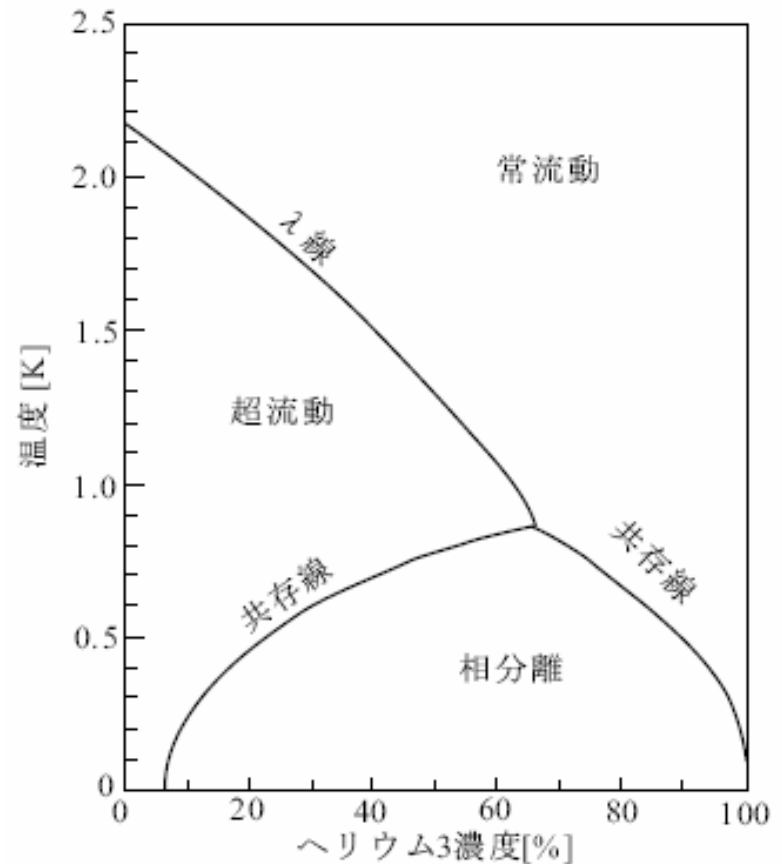
強制蒸発



希釈冷凍



$^3\text{He} - ^4\text{He}$ 混合液

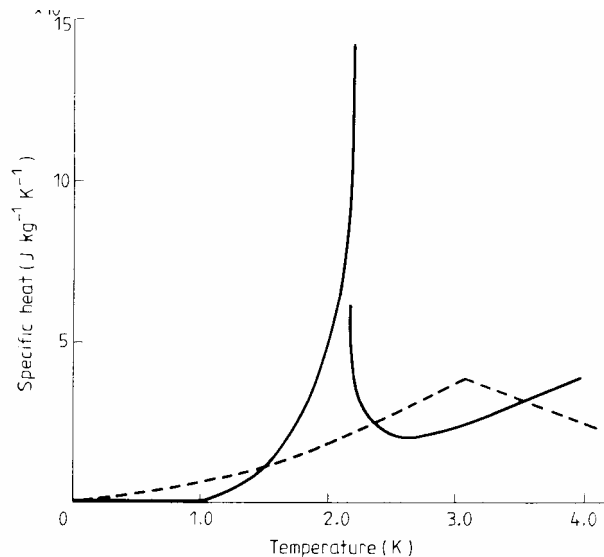
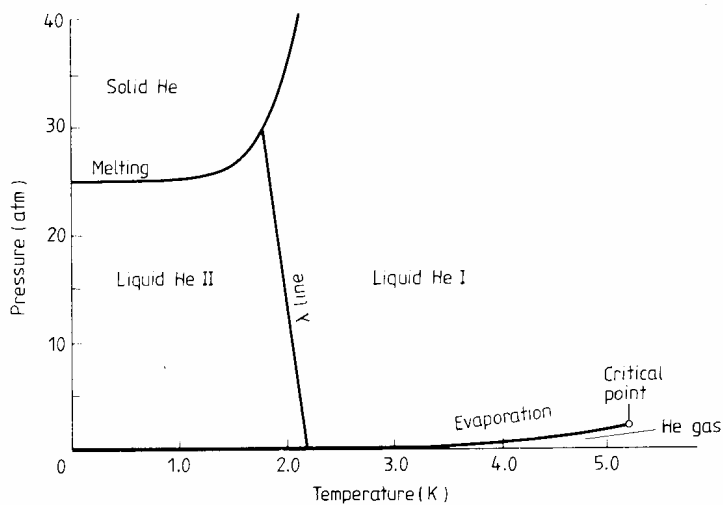


# 超流動転移

超流動転移 ( $T=2.17\text{K}$ )

He (常流動相)

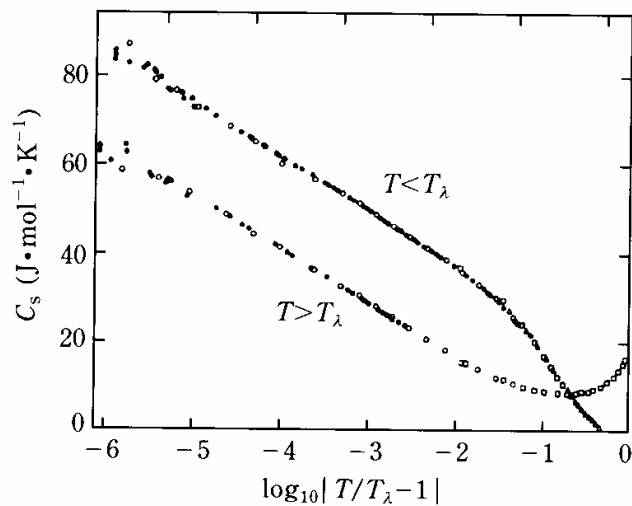
He (超流動相)



比熱の異常： 転移

臨界現象の研究

$$\Delta C \propto \log|T - T_\lambda|$$



# $^4\text{He}$ と $^3\text{He}$ の相図

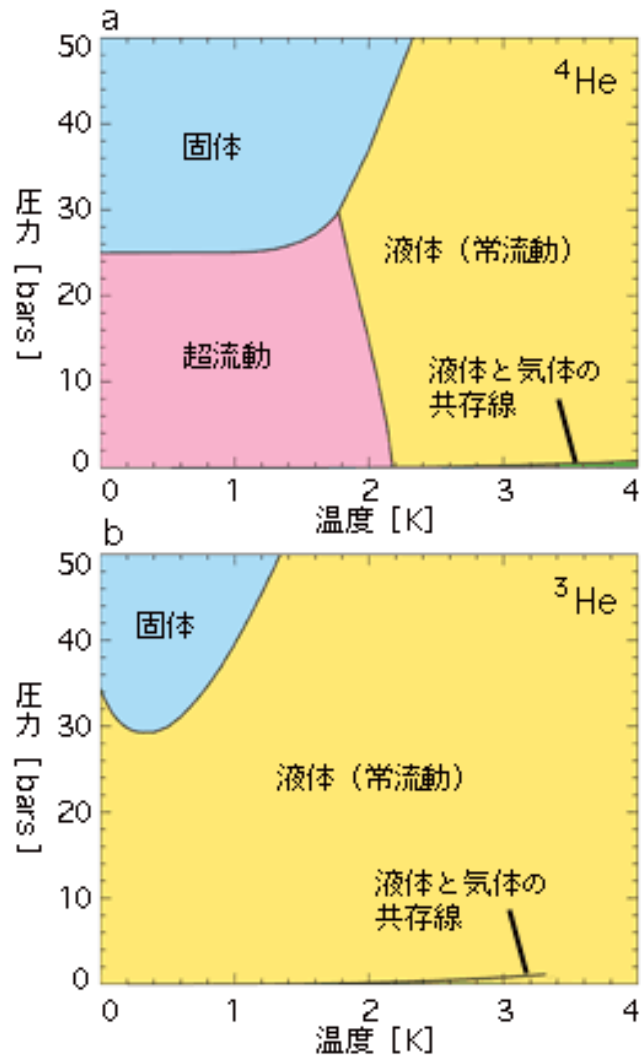


図 1 : ヘリウムの相図 [4K 以下]

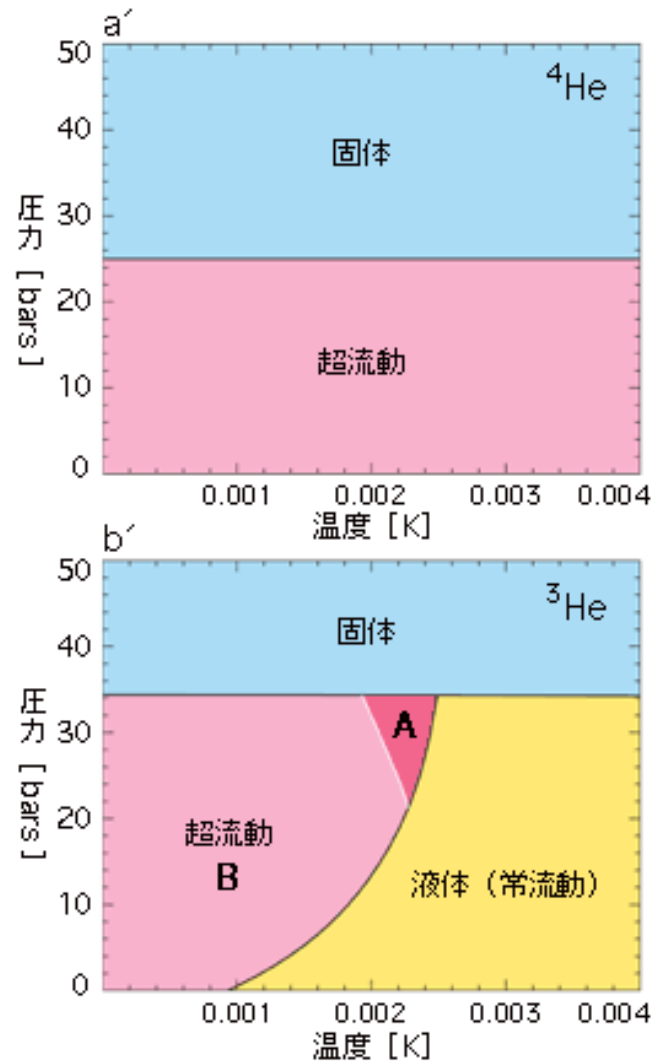
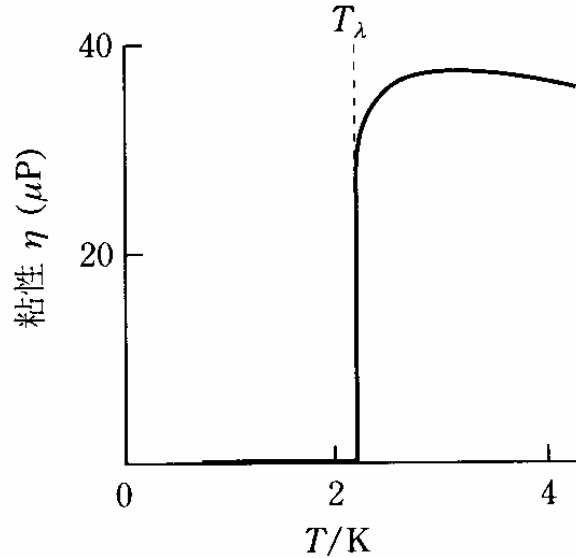
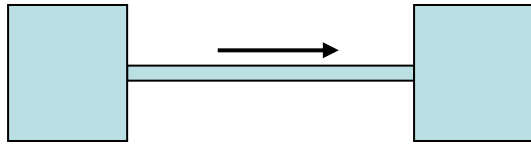
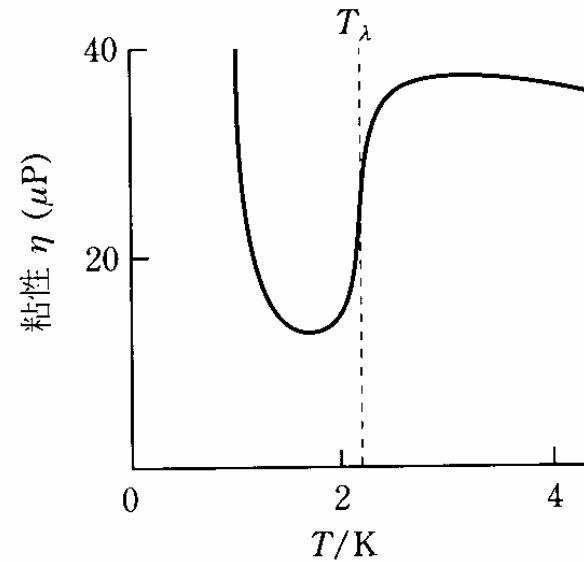
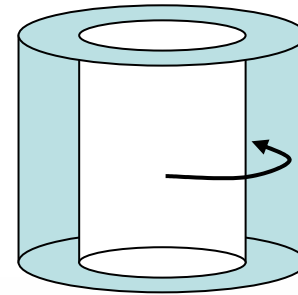


図 2 : ヘリウムの相図 [0.004K 以下]

# 超流動ヘリウム：粘性

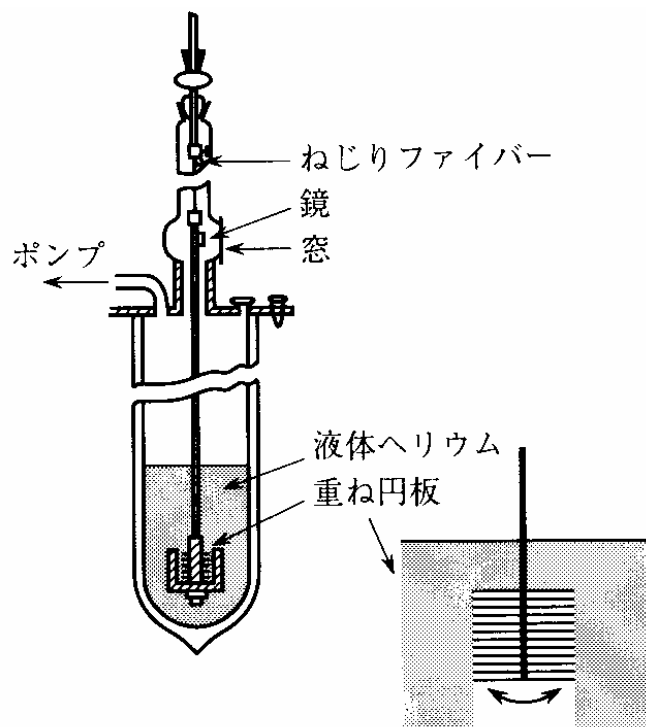


細管中の流れの圧力勾配の測定から求めた粘性



同心回転円筒型粘性計によって測定した粘性

# Andronikashviliの実験



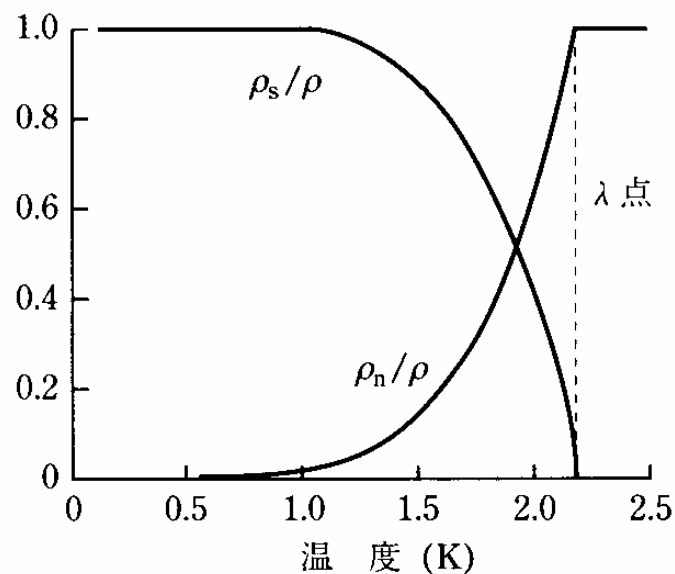
$$\lambda_\eta = \left( \frac{2\eta}{\rho_n \omega} \right)^{1/2}$$

よりも円板の間隔が十分に狭ければ常流体はすべて円板とともに動く

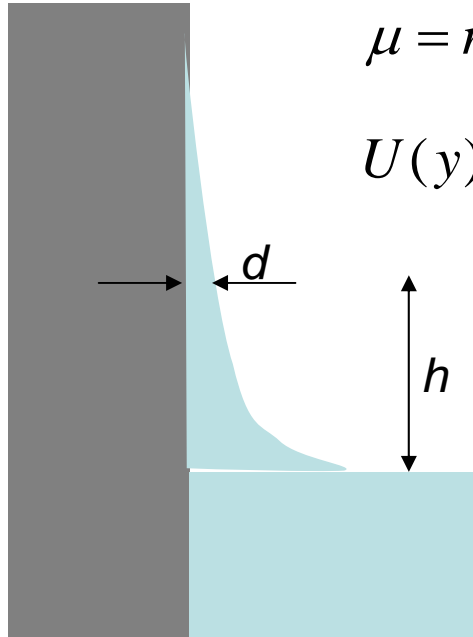
## 2流体モデル

$$\rho = \rho_s + \rho_n$$

$$\mathbf{j} = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n$$



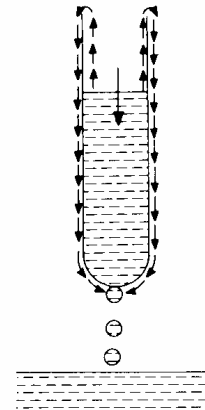
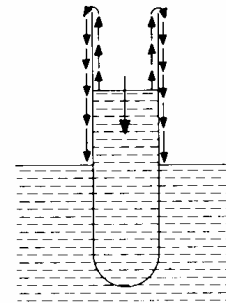
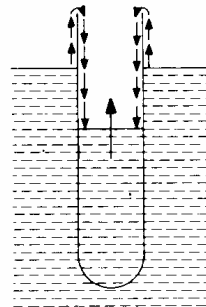
# ヘリウム膜 film flow



$$\mu = mgz - U(y)$$

$$U(y) = m \frac{\alpha}{y^n} \quad n \approx 3-4$$

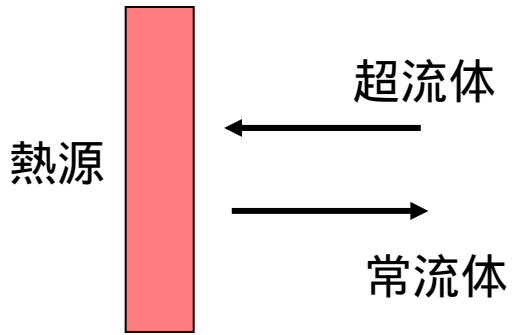
$$d(h) = \left( \frac{\alpha}{gh} \right)^{1/n}$$



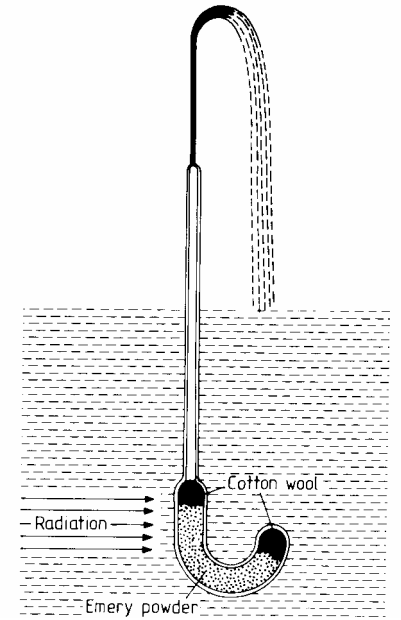
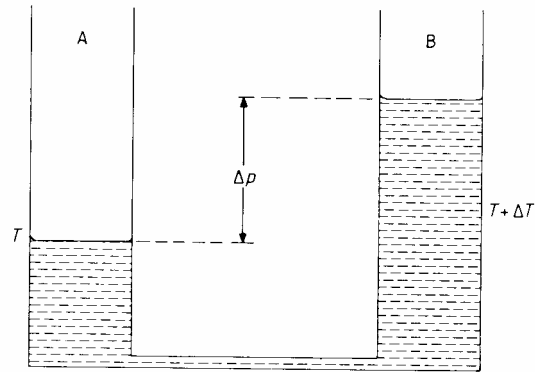
film flow



# 内部对流



# 熱機械效果 (噴水效果)



# 2流体モデルにおける音波

$$\rho_s \left( \frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla) \mathbf{v}_s \right) + \rho_n \left( \frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla) \mathbf{v}_n \right) = -\nabla p + \eta_n \nabla^2 \cdot \mathbf{v}_n$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n) = 0$$

$$\frac{\partial(\rho s)}{\partial t} + \nabla \cdot (\rho_s s \mathbf{v}_s + \rho_n s \mathbf{v}_n) = 0$$

第1音波

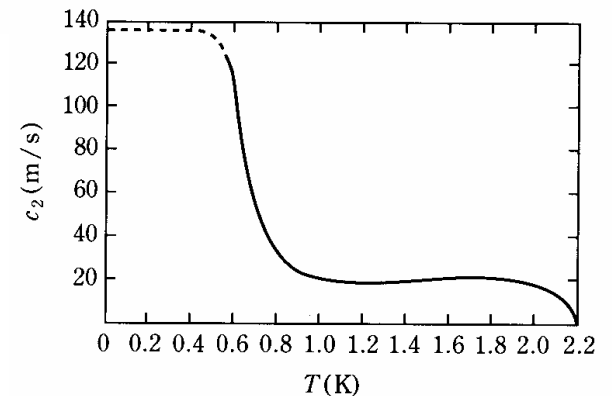
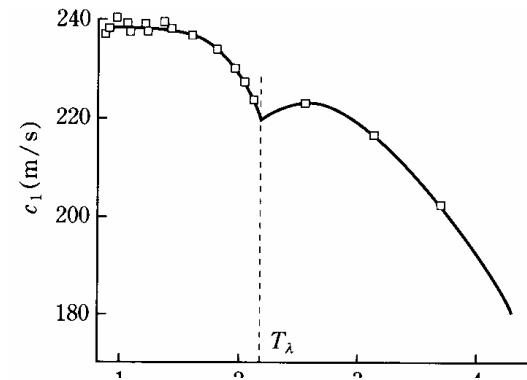
$$\frac{\partial^2 \delta \rho}{\partial t^2} = \nabla^2 \delta p$$

$$c_1^2 = \left( \frac{\partial p}{\partial \rho} \right)_s = \frac{1}{\rho_0 \kappa_s}$$

第2音波

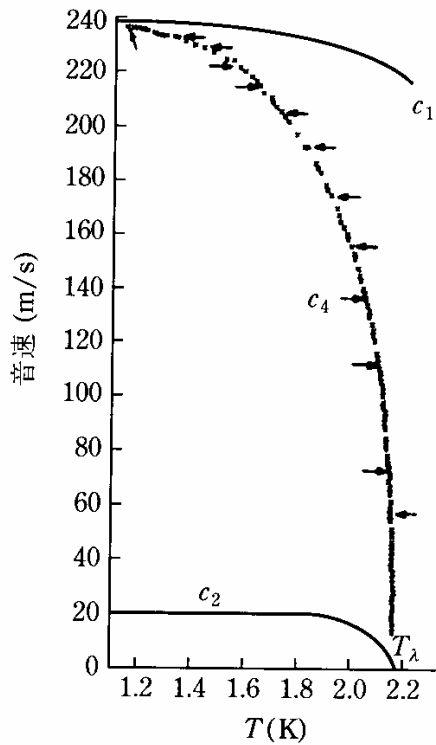
$$\frac{\partial^2 \delta s}{\partial t^2} = \frac{\rho_{s,0}}{\rho_{n,0}} s_0^2 \nabla^2 \delta T$$

$$c_2^2 = \frac{\rho_{s,0} s_0^2}{\rho_{n,0}} \left( \frac{\partial T}{\partial s} \right)_\rho = \frac{\rho_{s,0} s_0^2}{\rho_{n,0}} \frac{T}{C_V}$$



## 第4音波

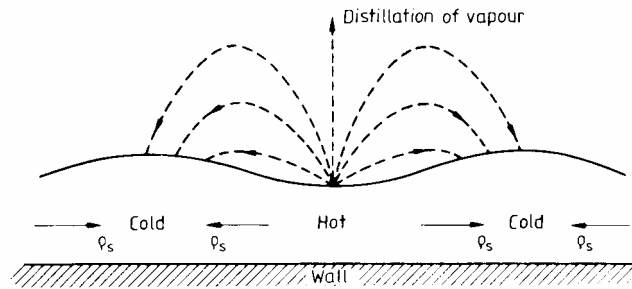
$$c_4^2 = \frac{\rho_s}{\rho} c_1^2 + \frac{\rho_n}{\rho} c_2^2$$



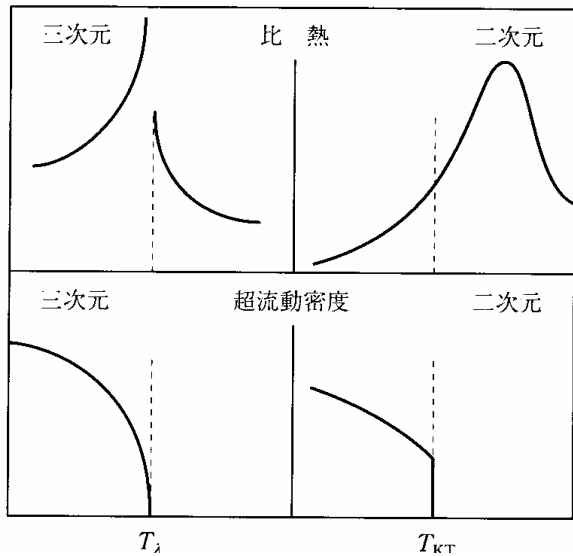
## 第3音波

$$\frac{\omega^2}{k^2} = \left( \frac{f}{k} + \frac{\sigma}{\rho} k \right) \tanh(kd) \Rightarrow \frac{\omega^2}{k^2} \approx fd \quad (k \rightarrow 0)$$

$$c_3^2 = \frac{\rho_s}{\rho} fd$$



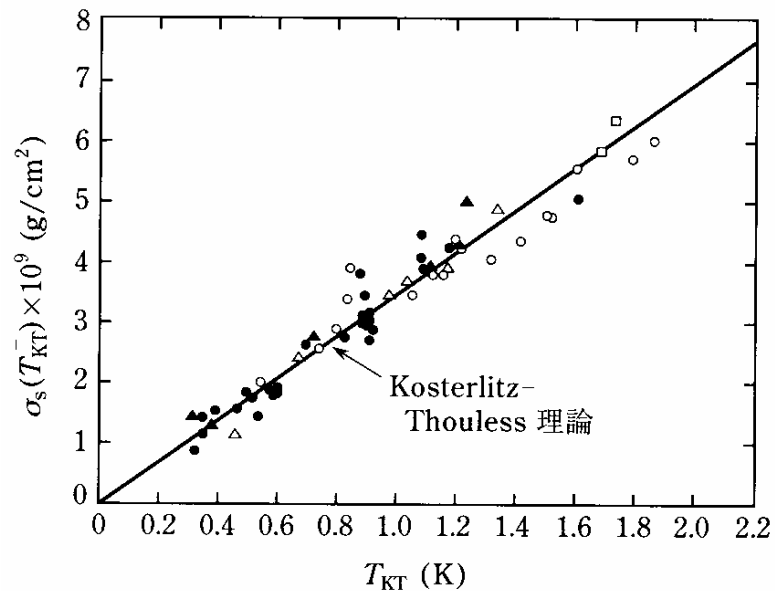
# ヘリウム薄膜の超流動転移



$$F = \pi K_0 \ln\left(\frac{R}{a}\right) - k_B T \ln\left(\frac{R}{a}\right)^2$$

$$k_B T_{KT} = \frac{\pi K_0}{2}$$

$$\frac{\rho_s(T_{KT})}{k_B T_{KT}} = \frac{2 m^2}{\pi \hbar^2}$$



# 低温物理学講義

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## 超流動 - その2

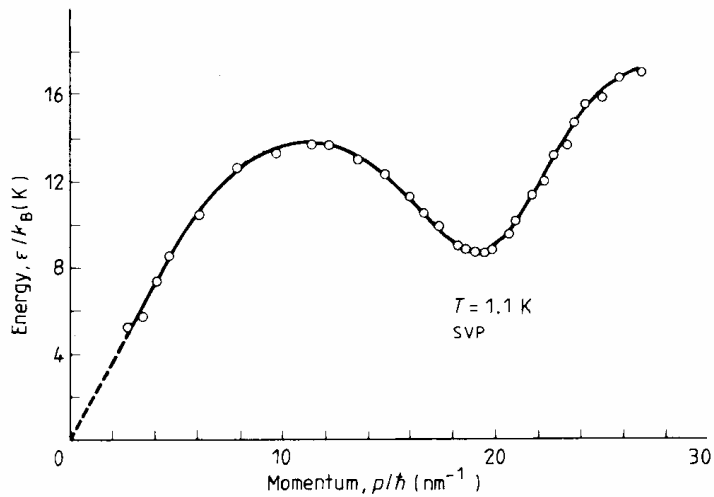
- 臨界速度, ランダウ条件
- フォノンとロトン
- 循環, 渦, 渦輪, 回復距離
- ヘリウム中のイオン, 移動度
- 渦とイオン
- ヘリウム液面電子系, 移動度, リップロン, ウィグナー結晶, デンプル

# フォノンとロトン

$$\varepsilon_p = \begin{cases} cp & p \approx 0 \\ \Delta + \frac{(p - p_0)^2}{2\mu} & p \approx p_0 \end{cases}$$

ランダウ条件

$$V_c = \left[ \frac{\varepsilon_p}{p} \right]_{\text{Min}}$$



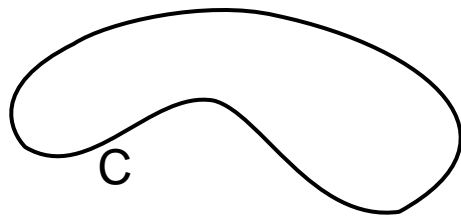
# 超流動

$$\Psi = |\Psi| \exp(i\theta)$$

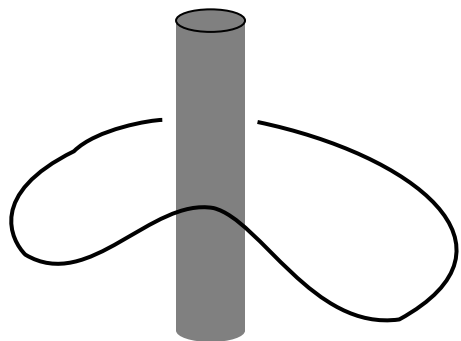
$$\nabla \times \mathbf{v}_s = 0 \quad \text{irrotational flow}$$

$$\mathbf{v}_s = \frac{\hbar}{m} \nabla \theta$$

$$\kappa \equiv \oint_C \mathbf{v}_s \cdot d\mathbf{s} \quad \text{循環 (circulation)}$$

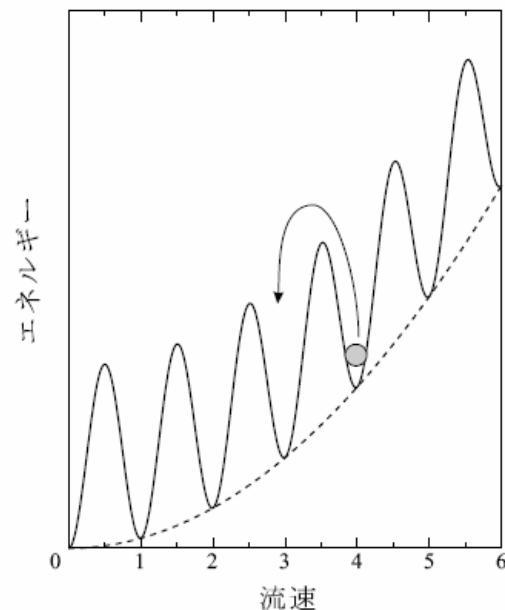
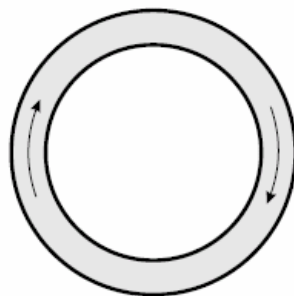


$$\kappa \equiv \oint_C \mathbf{v}_s \cdot d\mathbf{s} = 0$$

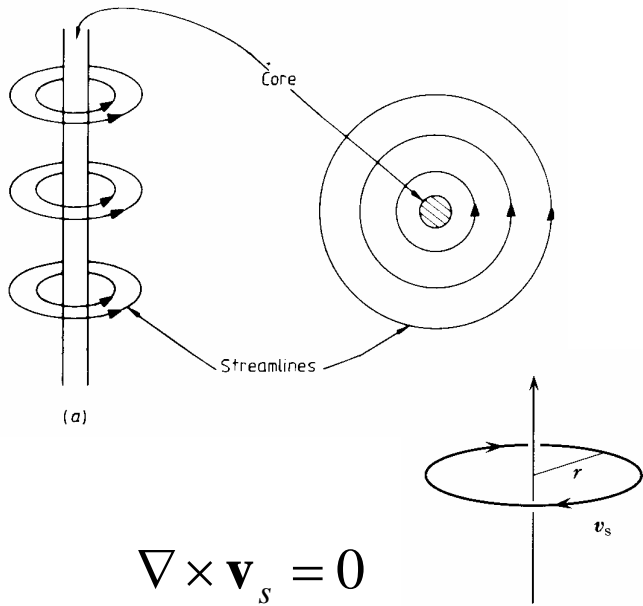


$$\kappa \equiv \oint_C \mathbf{v}_s \cdot d\mathbf{s} = \frac{\hbar}{m} \oint_C \nabla \theta \cdot d\mathbf{s} = \frac{\hbar}{m} 2n\pi = n \frac{h}{m}$$

永久流



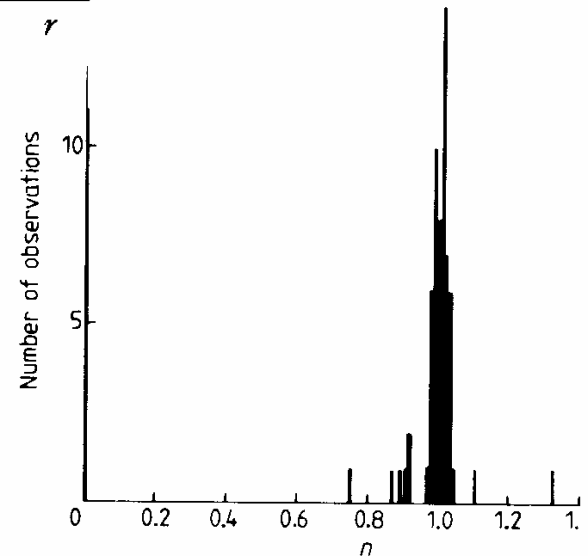
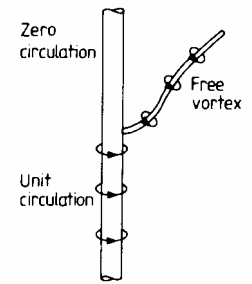
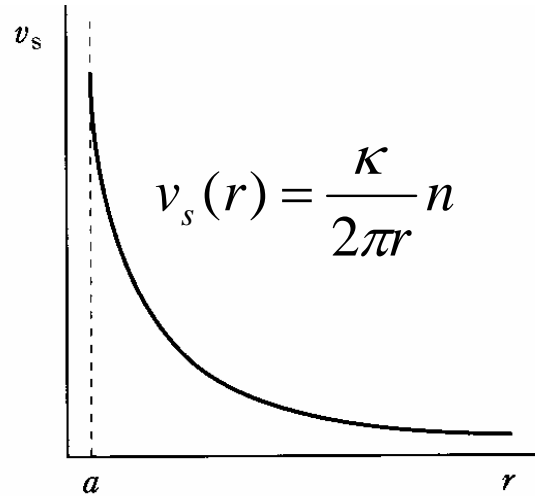
# 超流動量子渦



$$\nabla \times \mathbf{v}_s = 0$$

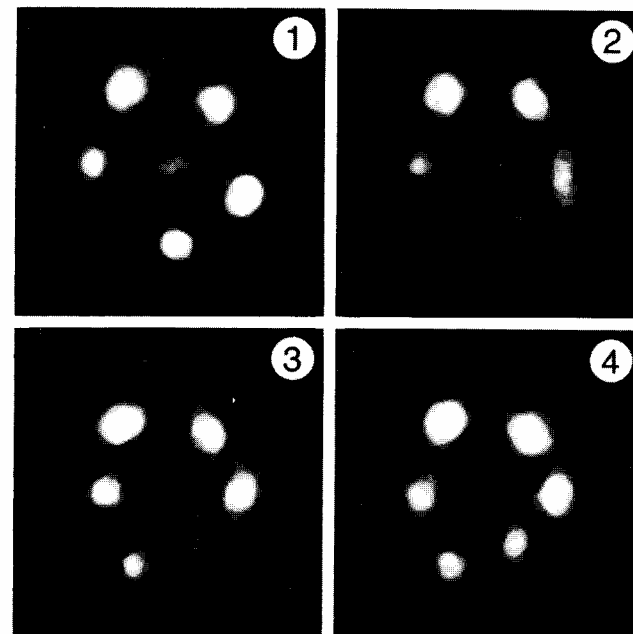
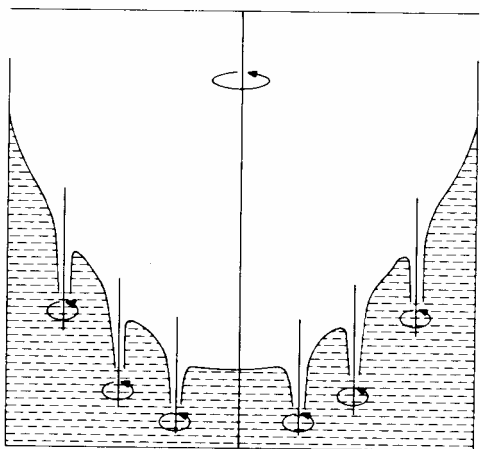
$$\kappa \equiv \oint_C \mathbf{v}_s \cdot d\mathbf{s} = \frac{\hbar}{m} \oint_C \nabla \varphi \cdot d\mathbf{s} = \frac{\hbar}{m} 2\pi n = n \frac{h}{m}$$

$$\frac{h}{m} = 0.998 \times 10^{-3} \text{ cm}^2/\text{s}$$





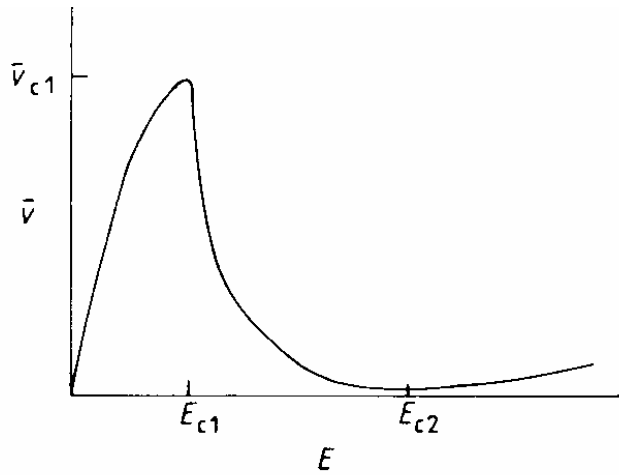
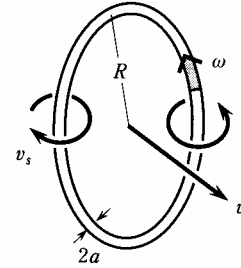
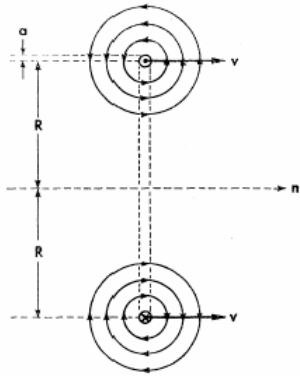
# 回転バケツの実験



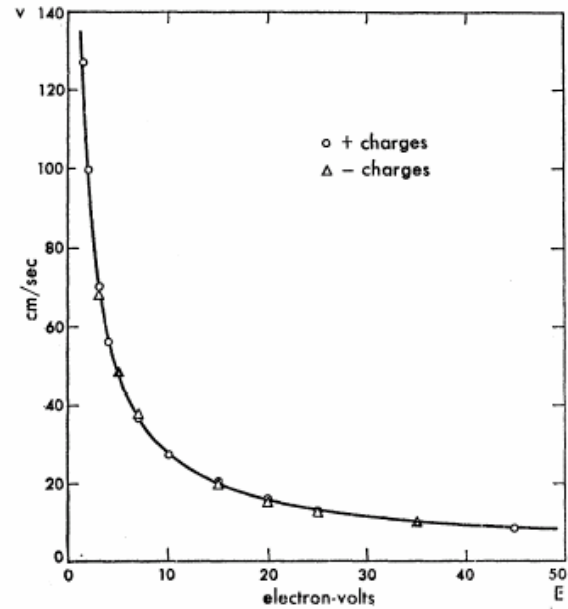
# 渦輪

$$v = \frac{\kappa}{4\pi R} \left[ \ln \frac{8R}{a} - \frac{1}{4} \right]$$

$$E = \frac{1}{2} \rho \kappa^2 R \left[ \ln \frac{8R}{a} - \frac{7}{4} \right]$$

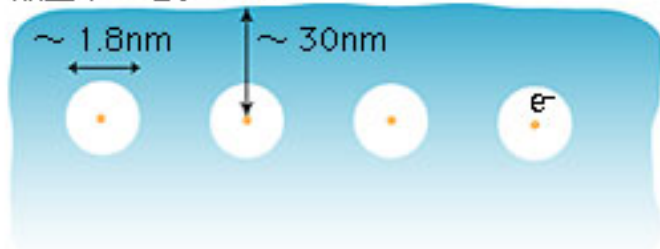


(b)



# 液体ヘリウム中のion

液面下の電子バブル



液面下のイオン (スノーボール)

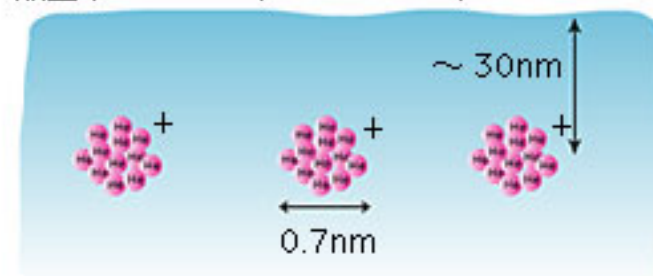
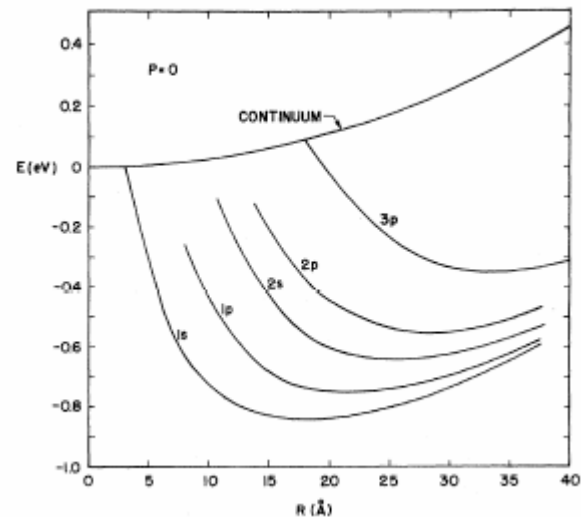


図 6: 液体ヘリウム表面の内側に打ち込まれた電子バブルとスノーボール

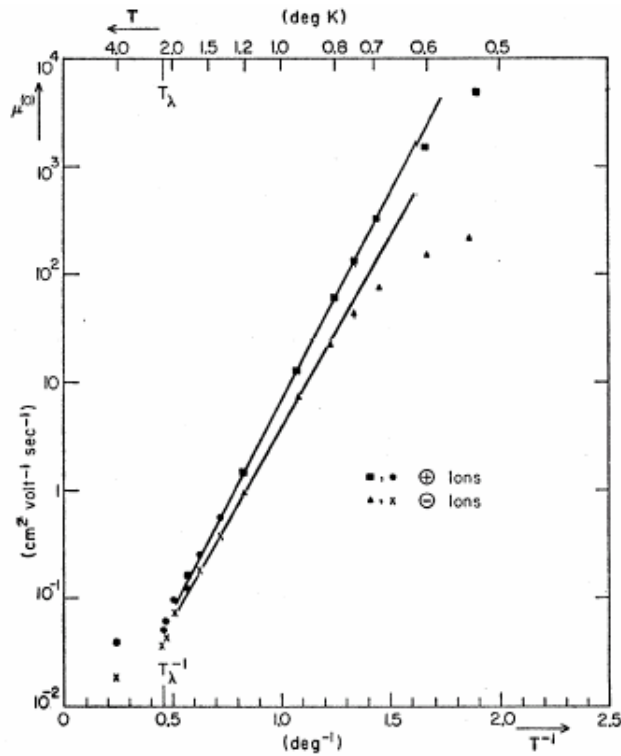
$$E = E_{e,j} + \frac{4\pi R^3}{3} p + 4\pi R^2 \sigma - \frac{1}{2} \frac{\epsilon - 1}{\epsilon} \frac{e^2}{R}$$

$$E = \frac{\hbar^2}{2m} \frac{1}{R^2} + 4\pi R^2 \sigma$$

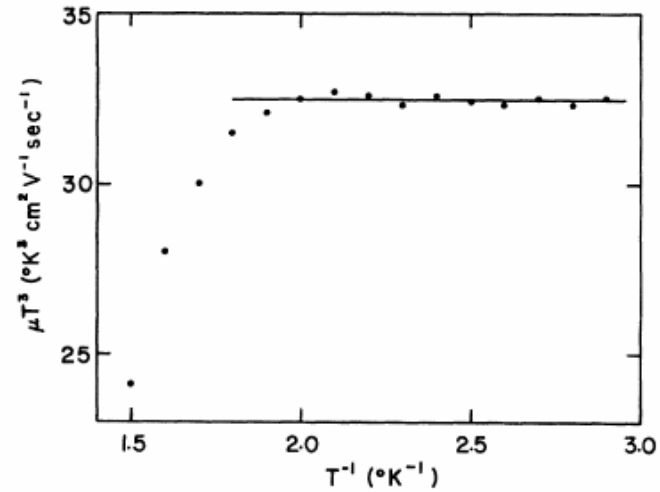


# 液体ヘリウム中のイオンの移動度

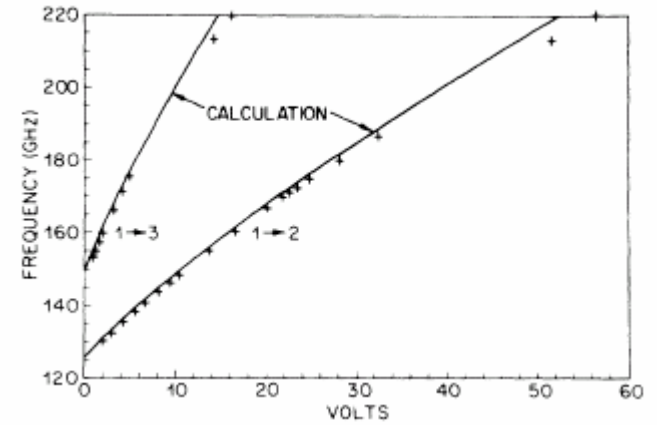
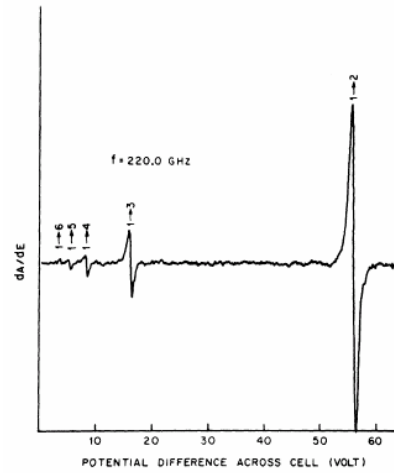
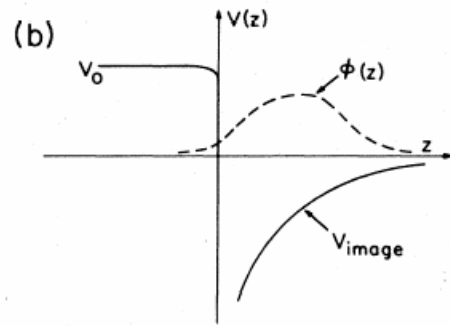
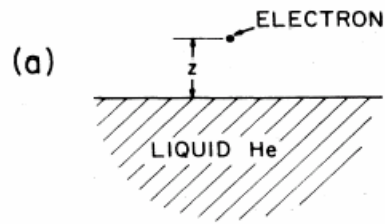
$$\mu_{\text{roton}} \propto e^{-\Delta/k_B T}$$



$$\mu_{\text{phonon}} \propto T^{-3}$$



# ヘリウム液面電子系



# 液面電子の移動度

$$\Gamma = \frac{e^2 n^{1/2}}{k_B T}$$

