I. INTRODUCTION

A two-dimensional electron gas (2DEG) under unidirectional potential modulation—a lateral superlattice (LSL)—is well known to show oscillatory magnetoresistance (Weiss oscillation) as a consequence of commensurability between the cyclotron diameter $2R_c = 2h k_F/|eB|$ and the period $a$ of the LSL, where $k_F = \sqrt{2\pi n_i}$ denotes the Fermi wave number, with $n_i$ the areal density of the 2DEG. Quantum-mechanical theories treating the modulation $V(x) = V_0 \cos(2\pi nx/a)$ as a first-order perturbation were developed by several authors. The theories show that the main contribution to the magnetoresistance oscillation results from the “band conductivity”; the width of the Landau levels, lifted from degenerated Landau levels, oscillates with $\partial E_x\partial k_y$, where $E_{N,k_y} = (N,k_y)\cos(x)$ of the $N$th Landau level and hence of the conductivity $\sigma_{xy}$. The resistivity $\rho_{xx}$ $= \sigma_{xx}/\sigma_{yy}$ oscillates accordingly. Resistivity minima occur at the condition when the Landau band collapses (flatband condition), given by

$$2R_c/a = n - \frac{1}{4} \quad (n = 1, 2, 3, \ldots). \quad (1.1)$$

Peeters and Vasilopoulos gave an asymptotic expression for the oscillatory part of the magnetoresistance, valid if the Landau quantum number $N$ is large enough at the Fermi energy $E_F = \pi\hbar^2 n_i/m^*$, with $m^*$ the electron effective mass (a condition fulfilled at low magnetic fields where Weiss oscillation is actually observed):

$$\Delta \rho_{xx}^{osc} = \frac{\eta^2}{\rho_0} \frac{L}{a} \frac{B}{\mu A} \left(\frac{T}{T_a(B)}\right)^2 \sin \left(2\pi\frac{2R_c}{a}\right), \quad (1.2)$$

where $\eta = V_0/E_F$, $\mu$ is the mobility, $L = h k_F/|e| e$ is the mean free path, and $A(x) = x/\sinh(x)$. The thermal damping factor $A[T/T_a(B)]$ is determined by the ratio of temperature $k_B T$ to the energy $k_B T_a = [1/(2\pi^2)](a k_F/2)\hbar\omega_c$, with $\omega_c = eB/m^*$. The latter energy represents the energy spread, multiplied by $1/(2\pi^2)$, over which the values of $2R_c$ differ by $a$, so that the periodic structure is smeared. This is reminiscent of the expression that appears in the thermal damping of the Shubnikov–de Haas (SdH) oscillation, $k_B T_a = [1/(2\pi^2)](a k_F/2)\hbar\omega_c$. The latter dependence on $L$ is valid at lower $B$. The damping can be accounted for by an additional factor of the form $[\pi/(\mu_B B)]/\sinh[\pi/(\mu_B B)]$. The parameter $\mu_B$ is found to be proportional to the mean free path $L$ of the 2DEG, and the coefficient of proportionality increases with $a$. The magnitude of $\mu_B$, as well as its dependence on $a$, and the electron areal density $n_e$ is close to that of $\mu_0$, the mobility corresponding to the total scattering time.

II. EXPERIMENT

Lateral superlattices were prepared from several 2DEG wafers [conventional molecular-beam-epitaxy (MBE)-grown...
GaAs/Al_{x}Ga_{1-x}As single heterostructures] with varying \( \mu \) and \( d_{s} \). The parameters of the LSL’s measured are tabulated in Table I. The depth \( d \) of the heterointerface from the top surface includes a 10-nm GaAs cap layer, a 40-nm Al_{x}Ga_{1-x}As layer uniformly doped with Si \([(2–5) \times 10^{24} \text{ m}^{-3}] \), and an Al_{x}Ga_{1-x}As undoped spacer layer with thickness \( d_{s} \) for samples \( H \), \( M1 \), \( M2 \), \( C \), and \( L \). For samples \( S1 \) and \( S2 \), a specially designed shallow 2DEG with \( \delta \)-doped Si layer\(^8\) was employed. As shown in Fig. 1, two serial Hall bars were prepared on one device, onto one of which a grating made of a high-resolution electron beam (EB) negative resist (calixarene derivative\(^8\)) was placed to introduce potential modulation. The other Hall bar was used as a reference. With this procedure, a LSL with a period down to \( a = 70 \text{ nm} \), that shows a clear Weiss oscillation, was prepared. However, for small \( a \), the oscillation amplitude was too small to bear a reliable quantitative analysis. Therefore we limit our analysis to the results from \( a = 115 \text{ nm} \) LSL’s (samples \( H \), \( M1 \), \( L \), and \( C \)) in the following. The relatively small \( a \) allowed many oscillations up to high index \( n \) (typically \( n = 3–15 \)) to be observed. A potential modulation was brought about by differential contractions between the resist and the wafer itself when the device was cooled from room temperature down to 4.2 K, the temperature at which measurement was made. The strain, thus introduced, piezoelectrically couples to the 2DEG, and causes modulation in the 2DEG plane.\(^10\) In order to maximize the effect, the (110) direction was selected as the direction of modulation.\(^11\) Even so, the modulation amplitude was very small: as will be described later, \( V_{0} \) was around 0.05 meV, or less than 1% of \( E_{F} \). We attribute this to the small effects, the strain and/or Fermi energy pinning, of the resist we have chosen, and also to the small \( a \) to \( d \) ratio. A small \( V_{0} \) was quite favorable for validating a perturbative treatment of the modulation. By comparison with the reference Hall bar, we have verified through Hall and SdH measurements that the grating did not bring about any deterioration of \( \mu \) or change in (the average) \( n_{s} \), in spite of the very high dose required for the EB resist \([ \sim 7 \text{ mC cm}^{-2} \text{ (Ref. } 9 \text{)}] \).

The magnetoresistance measurement was carried out with a standard low-frequency ac technique at 4.2 K. Such a high temperature was deliberately chosen in order to kill the SdH oscillation in the field range \( (0.1–0.4 \text{ T}) \) of present interest.

### III. RESULTS

Figure 2(a) shows magnetoresistance of a LSL, sample \( H \), and of its unmodulated counterpart (control), measured after illumination by light. From these raw data, the oscillatory part is extracted in the following procedure. As a first step to eliminate the slowly varying background, the resistivity of the control sample was subtracted [Fig. 2(b)].\(^12\) Then the upper and lower envelope curves were found as spline curves tangential to the upper and lower bounds of the trace, respectively. The average curve of the two envelopes was sub-

### TABLE I. Parameters of lateral superlattices measured.

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>( n_{\text{min}} ) a</th>
<th>( \mu_{\text{min}} ) a</th>
<th>( n_{\text{max}} ) b</th>
<th>( \mu_{\text{max}} ) b</th>
<th>( d_{s} )</th>
<th>( d )</th>
<th>( a )</th>
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<tbody>
<tr>
<td>( H )</td>
<td>2.0</td>
<td>69</td>
<td>2.3</td>
<td>79</td>
<td>40</td>
<td>90</td>
<td>115</td>
</tr>
<tr>
<td>( M1 )</td>
<td>2.2</td>
<td>56</td>
<td>2.7</td>
<td>72</td>
<td>40</td>
<td>90</td>
<td>115</td>
</tr>
<tr>
<td>( L )</td>
<td>2.5</td>
<td>19</td>
<td>3.0</td>
<td>28</td>
<td>40</td>
<td>90</td>
<td>115</td>
</tr>
<tr>
<td>( C )</td>
<td>2.6</td>
<td>34</td>
<td>5.1</td>
<td>101</td>
<td>20</td>
<td>70</td>
<td>115</td>
</tr>
<tr>
<td>( M2 )</td>
<td>2.2</td>
<td>66</td>
<td>2.7</td>
<td>85</td>
<td>40</td>
<td>90</td>
<td>105</td>
</tr>
<tr>
<td>( S1 )</td>
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<td>42</td>
<td>-</td>
<td>-</td>
<td>11.4</td>
<td>25</td>
<td>80</td>
</tr>
<tr>
<td>( S2 )</td>
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<td>17</td>
<td>-</td>
<td>-</td>
<td>11.4</td>
<td>25</td>
<td>70</td>
</tr>
</tbody>
</table>

*aMeasured in the dark.

*bMaximum values measured after successive illumination.

---

FIG. 1. Schematic diagram of a device with a scanning electron microscopy image of the grating \( (a = 70 \text{ nm}) \). Darker areas represent the resist, and brighter areas the bare GaAs surface. Bright lines are the edges of the resist.
illumination, did not display any noticeable deviation from Eq. 1.2 with \( V_0 = 0.041, 0.050, \) and \( 0.045 \) meV, respectively. The values of \( V_0 \) shown here might possibly be underestimating the modulation amplitude by factor of roughly 2. See Sec. IV. In contrast to what was reported so far, we have shown that Eq. 1.2 can, under a certain condition, correctly reproduce the experimental trace. It is worth pointing out here the important role played by the factor \( \frac{A}{T_a(B)} \). This factor was often neglected in semiclassical theoretical treatments, 6 including those published recently, 13,14 since they considered, at least approximately, only \( T=0 \). This factor is also not taken into account in experimental papers concerned with the envelope of Weiss oscillation. 15–17 The validity of the factor, nevertheless, was experimentally verified by Beton et al., 7 even before the paper by Peeters and Vasilopoulos, 5 by measuring the temperature dependence of the oscillation amplitude with a fixed \( B \). [Their \( B \) dependence, however, did not follow Eq. (1.2).] When \( ak_f/2 \) is large and (therefore) \( T_a(B) \) is much larger than the measuring temperature \( T \), \( A[T/T_a(B)] \approx 1 \), allowing the factor to be ignored. But since our present LSL’s have relatively small \( a \) and \( n_s \), \( ak_f \) is not so large. For the measurement shown in Fig. 2, \( ak_f/2 = 6.8 \); hence \( T_a(B) = 6.9B \) (in T) K, which is even smaller than \( T=4.2 \) K at 0.1–0.4 T. As a result, \( A[T/T_a(B)] \) is much smaller than unity and has a strong \( B \) dependence, resulting from the \( B \) dependence of \( T_a(B) \), at the magnetic field range of interest [see Fig. 2(d)]. It is obvious that without the factor, an experimental damping of the oscillation has not been reproduced. Our measurement sheds light on the importance of the factor \( \frac{A}{T_a(B)} \), and also reconfirms the validity of the factor from a viewpoint different from that of Beton et al.

Although we have seen that Eq. 1.2 describes experimental traces of the Weiss oscillation very well, we cannot expect this to be true regardless of the quality of the LSL. Theories 5,6 did not take into account the collision of electrons that scatters electrons away from the cyclotron orbit before it completes a cycle. Therefore, the theories apply only for LSL’s with high enough mobility so that the mean free path is much longer than the cyclotron circumference. In fact, our sample \( L \), the LSL fabricated from a 2DEG with low mobility, showed a deviation from Eq. (1.2), as shown in Fig. 3(a). The growth profile of the 2DEG wafer, including the thickness of the spacer layer \( (d_s) \), the silicon-doped
The result of the fitting is also displayed in the insets, showing the ratio of 1/sinh(maxima) to samples H and M1. We tried fitting to the function \( P_1/\sinh(P_2/B) \), with \( P_1 \) and \( P_2 \) as fitting parameters. The result of the fitting is also displayed in the insets, showing reasonable agreement with the data.\(^{19}\) The value \( \mu_w \) = \( \pi/P_2 \) obtained from the fitting, however, is much smaller than \( \mu \) calculated from the zero-field resistivity \( \rho_0 \): \( \mu_w = 6.1 \text{ m}^2/\text{V s} \) and 4.8 \( \text{m}^2/\text{V s} \), to be compared with \( \mu = 24 \text{ m}^2/\text{V s} \) and 62 \( \text{m}^2/\text{V s} \) for Figs. 3(a) and 3(b), respectively. Note the difference in the ratio \( \mu_w/\mu \); the ratio for the latter is much smaller. This will be discussed later. Thus, to describe our measurement, Eq. (1.2) needs to be modified to include another damping factor \( A[\pi/(\mu_wB)] \) as

\[
\frac{\Delta \rho_{\text{osc}}}{\rho_0} = A \left( \frac{\pi}{\mu_wB} \right) \frac{\eta^2 L}{2} B \mu A \left( \frac{T}{T_a(B)} \right) \sin \left( 2 \pi \frac{2R}{a} \right).
\]

(3.1)

As shown in Fig. 3, Eq. (3.1) reproduces the experimental traces. When \( \mu_w \) is large, \( A[\pi/(\mu_wB)] \) tends to unity; for \( \mu_w \) greater than about 20 \( \text{m}^2/\text{V s} \), Eq. (3.1) is practically indistinguishable from Eq. (1.2) at 4.2 K. Samples H and M1 are also described by Eq. (3.1) with large enough \( \mu_w \). Therefore Eq. (3.1) is not inconsistent with the observation of Fig. 2.

To see the dependence of our damping parameter \( \mu_w \) on the mobility \( \mu \), we successively illuminated sample C with an infrared light-emitting diode, and gradually increased \( n_s \) and \( \mu \). The evolution of oscillation envelope is shown in Fig. 4. As can be seen, the experimental trace becomes progressively closer to Eq. (1.2) with the increase of \( \mu \); \( \mu_w \), that gives the best fit to Eq. (3.1), increases with \( \mu \). It is important to point out that the second and third traces, which show clear deviations from Eq. (1.2), have comparable, even higher mobilities than samples H and M1. Apparently \( \mu_w \) is not determined solely by \( \mu \).

The values of \( \mu_w \) giving the best fit are plotted in Fig. 5 for the four samples as functions of the mean free path \( L = h k_F \mu/e \). As can be readily seen, the plots can be divided into two groups, one with LSL’s prepared from a 2DEG with \( d = 40 \text{ nm} \) (samples H, M1, and H), and the other from a 2DEG with \( d = 20 \text{ nm} \) (sample C). The error bars in the figure represent the uncertainty of the fitting to the function \( P_1/\sinh(P_2/B) \). For large \( \mu_w \), i.e., for small \( P_2 \), the function tends to \( P_1 B/P_2 \). It then becomes difficult to determine \( P_1 \) and \( P_2 \) independently, resulting in the large error bars shown. Within each group, \( \mu_w \) is seen to be nearly proportional to \( L \). The coefficient is about four times larger for the first group. The ratio is, probably fortuitously, the same as \( d_2^2 \). Therefore, the values of \( \mu_w \) replotted as a function of \( k_F d_2^2 \mu \) fall on a single line (numerically approximately \( \mu_w = 1/(2 \pi^3 a_{\text{B}}^2) k_F d_2^2 \mu, \) with \( a_{\text{B}}^2 = 10.2 \text{ nm} \) the effective Bohr radius in GaAs.) More significantly, the magnitude of \( \mu_w \) is close to that of \( \mu_Q \), the quantum mobility, obtained from a Dingle analysis of the experimental SdH traces. This implies that small-angle scattering plays an important role in the damping of the Weiss oscillation, as will be discussed in Sec. IV.

IV. DISCUSSION

It is well known that for a GaAs/Al{\text{subscript}2}Ga{\text{subscript}1-x}As 2DEG, the (transport) mobility \( \mu = e \tau/m^* \) is often higher than the quantum mobility \( \mu_Q = e \tau_Q/m^* \) by an order of magnitude, where \( \tau \) and \( \tau_Q \) represent the momentum-relaxation time and the total scattering time, respectively. This is because small-
angle scattering by remote ionized donors, one of the main scattering processes in the system, contributes much less to the former. Nevertheless, it was pointed out by several authors\(^5\,14\) that small-angle scattering should be considered as a scattering that scatters electrons away from the cyclotron trajectory, and affects the amplitude of the Weiss oscillation. Without scattering, \(\Delta \rho_{xx}/\rho_0 \propto B\), apart from the \(B\) dependence of the factor \(A[T/T_d(B)]\) [see Eq. (1.2)]. Beton et al.\(^7\) suggested that an exponential factor should be included so that \(\Delta \rho_{xx}/\rho_0 \propto B \exp[-\pi(\mu B)]\), with their experimental \(\mu\) consistent with \(\mu_Q\). Paltiel et al.\(^16\) on the other hand, proposed a \(B \exp(-B_a^2/B^2)\) dependence which explained their experiment (and also recent experiment by Long et al.\(^13\)) well. These formulas are more or less of empirical nature, multiplying additional damping factor to \(B\). A more rigorous treatment of Boltzmann equation by Mirlin and Wölfle,\(^14\) however, showed that the factor \(B\) should be removed from these formulas. They showed, as mentioned earlier, that the factor that should be included is \([\pi(\mu B)]/\sinh(\pi(\mu B))\) for an isotropic scattering model, a model that does not take the difference between \(\mu\) and \(\mu_Q\) into account. In a more realistic long-range random scattering model, they showed that the factor is modified by replacing \(\mu\) with \(\mu^* = \mu_Q/\{1 - \{1 + \mu_Q/(\mu L/a)^2\}^{1/2}\}\). Its approximate formulas can be expressed as follows: at low field \(B < B_2 = \pi(L/a)^2\mu_Q/2\), a similar formula to that of Beton et al., but without the factor \(B\); in the middle field range \(B_2 < B < B_1 = \pi(4L/a)^2/(2\mu)\), \(\Delta \rho_{xx}/\rho_0 \propto \sinh(\pi(\mu B))\), similarly to Paltiel et al., but again without \(B\). The erroneous inclusion of \(B\) can lead to a factor of 2–3 overestimation of \(\mu_Q\) in the low-field case.

Returning to our experiment, \(B_1\) and \(B_2\) (using \(\mu_w\) in place of \(\mu_Q\)) in the above calculation fall in between 0.7–1.2 and 0.9–2.5 T, respectively. Both fields are relatively large owing to large values of \(L/a\), and for most of the samples \(B_2\) is even larger than \(B_1\). The field 0.1–0.4 T where the Weiss oscillation is observed may, therefore, be classified in the aforementioned low-field regime. Thus the oscillation amplitude is, according to the theory by Mirlin and Wölfle, expected to be proportional to \(1/\sinh(\pi(\mu B))\), implying that our \(\mu_w\) equals \(\mu_Q\). This is not inconsistent with the observation in Fig. 5: \(\mu_w\) and \(\mu_Q\) are in reasonable agreement, although a discrepancy is seen, especially for sample \(C\) in the intermediate mean free path. We believe the discrepancy to result mainly from the limited validity of the values of \(\mu_Q\).

The Dingle analysis of the Shd oscillation has been known to be quite vulnerable to even a slight inhomogeneity of \(n_s\). The inhomogeneity manifests itself as a curvature in the Dingle plot and/or a deviation of the 1/\(B\rightarrow0\) intercept from the theoretical value ~4,~ owing to destructive interference of Shd oscillations with varying frequencies.\(^20\) The effect usually makes the slope of the Dingle plot appear larger; hence the resulting \(\mu_Q\) smaller. The degree of inhomogeneity...
ity can vary between wafers or between different illumination conditions. It is possible that after slight illumination, before the saturation of DX-center excitation, inhomogeneities become more pronounced. However, it was not possible to quantitatively estimate these and other effects which challenge the reliability of the values of $\mu_Q$, mainly because our experimental Dingle plot was taken from rather narrow field range of 0.5-0.9 T, where a SdH oscillation was observed at 4.2 K. Another point that suggests $\mu_W = \mu_Q$ is the dependence of $\mu_W$ on $d_z$. Coleridge\textsuperscript{20} showed that both $\mu$ and $\mu_Q$ and also the ratio $\mu/\mu_Q$ increase with $d_z$ for small $d_z$, experience maximum at a certain $d_z$, and then decrease. The value of $d_z$ that gives the maximum values depends on the background acceptor density $N_A$ in the GaAs channel. For the estimated $N_A$ for our 2DEG, $d_z = 20-40$ nm is still in the increasing regime $\mu_Q$. Therefore, $\mu_Q$ should be larger for $d_z = 40$ nm. Conversely, we postulate, the relation $\mu_W = \mu_Q$ then the measurement of Weiss oscillation damping may provide an alternative (probably more robust) method for determining $\mu_Q$.

Our finding of the damping factor $A(\pi/(\mu_w B))$ is in qualitative disagreement with Paltiel et al.\textsuperscript{16} and Long et al.\textsuperscript{17}; none of our traces show a reasonable fit to $B \exp (-B_0^2/B^2)$. The reason for this is not clear at present.\textsuperscript{21} At least in Ref. 16, the values of $n_s$, $\mu$, $\mu_Q$, and $a$ are similar to ours. Therefore both $B_1$ and $B_2$ are almost the same, categorizing the sample of Ref. 16 into the low-field regime of Mirlin and Wölfle. One possible explanation for the discrepancy is the difference in the modulation amplitude $V_0$. The modulation amplitudes of Refs. 16 and 17 are orders of magnitude larger than ours. The potential modulation is inevitably accompanied by a position-dependent electron density, and hence by a position-dependent $k_F$. The effect is not taken into consideration in perturbative calculations at all, but can affect the amplitude of the Weiss oscillation, especially in the lower field, as is the case with the SdH oscillation. However, it is beyond of the scope of the present paper to evaluate this effect.

Finally we address the issue of the magnitude $\eta = V_0/E_F$ of the potential modulation that can be deduced from our present analysis. The values of $\eta$ derived by fitting the experimental traces to Eq. (3.1) are plotted against the electron areal density $n_s$ in Fig. 6; actually $\eta$ was obtained by fitting the function $P_1/\sinh(P_2/B)$ to the plots of the extreme of $\Delta p_{osc}/(p_0 A[T/T_0(B)])$ (see the inset of Fig. 3), with $\eta = \sqrt{2}aP_1/(L\mu P_2)$. For each sample, $\eta$ decreases with $n_s$ faster than $n_s^{-1/2}E_F^{-1}$. This presumably reflects the increase of screening, which diminishes the efficacy of the perturbation brought about by the grating. Prior to the present study, $\eta$ was usually obtained by using Eq. (1.2), often neglecting the factor $A[T/T_0(B)]$, and by picking up the oscillation amplitude of the lowest index $n$, i.e., the highest field (see, e.g., Ref. 17). The values of $\eta$ thus obtained are identical to ours provided that the damping is completely negligible ($A[T/T_0(B)] = A(\pi/(\mu_w B)) = 1$), the condition usually not fulfilled. In general, using Eq. (3.1) instead of Eq. (1.2) has the advantages of (1) taking into account the damping that has already occurred even at the lowest index, and (2) obtaining $\eta$ is common to all the indices $n$. However, the possibility that the present treatment still underestimates the value of $\eta$ cannot be completely ruled out. In obtaining Eq. (3.1), we rather arbitrarily just multiplied the Eq. (1.2) by factor $A(\pi/(\mu_w B))$ as a natural extension of the equation. Although this procedure successfully explains the $B$ dependence of the oscillation modified by the (small-angle) scattering, it might be argued that the scattering also reduces the amplitude by multiplying Eq. (1.2) by a factor independent of $B$. In fact, the theory of Mirlin and Wölfle requires the inclusion of another factor $\mu_Q/\mu$ into Eq. (3.1) (with $\mu_w$ replaced by $\mu_Q$ in the equation). Identifying our $\mu_w$ with $\mu_Q$, the resultant amplitude should be altered from $\eta$ to $\eta' = \eta\sqrt{\mu_w/\mu}$. Numerically, the correction factor is roughly 2 for samples $H$, $M1$, and $L$, and around 3 for sample $C$. Another independent way to estimate $\eta$ is desired to know which equation is the correct one. Positive magnetoresistance (PMR) at the low fields\textsuperscript{22} is often used for this purpose. Unfortunately, owing to the smallness of the modulation amplitude, PMR was very small, sometimes totally unobservable, for our present samples, and therefore cannot be used for a reliable analysis. Our recent experiment using magnetic LSL with controllable modulation amplitude\textsuperscript{23} suggests, however, that $\eta'$ overestimates the amplitude by comparison with the modulation amplitude estimated from PMR.

V. CONCLUSIONS

We have shown that Eq. (3.1) reproduces the oscillatory part of the magnetoresistance very well. The parameter $\mu_w$ was found to be proportional to $L$, and numerically was $\mu_w \approx 1.6 \times 10^{-3} k_F d_z^2 \mu$ (with $k_F$ in nm\textsuperscript{-1} and $d_z$ in nm). For large enough $\mu_w$, Eq. (3.1) is indistinguishable from Eq. (1.2). Comparison of the damping factor with recent theory\textsuperscript{14} suggests $\mu_w = \mu_Q$, which is not inconsistent with our experimental $\mu_Q$. This implies that scattering events, regardless of the scattering angle, contribute to the damping of the Weiss oscillation. To establish a more precise relation between $\eta$ and the oscillation amplitude, it might be necessary to include an additional constant factor in Eq. (3.1), which is a problem that requires further study.

ACKNOWLEDGMENTS

This work was supported in part by a Grant-in-Aid for Scientific Research (10740142) from the Ministry of Education, Science, Sports, and Culture, and also in part by a grant from the Foundation Advanced Technology Institute.
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12 It is not necessary to follow this first step. Although fabricated within a single chip, the reference Hall bar is a few hundred micrometers away from the LSL. The slowly varying background, the origin of which is not exactly known in general, can be severely dependent on the local condition, impurity distribution etc., of the 2DEG. The control Hall bar provides only rough estimation of the background for the LSL. Its subtraction helps to find upper and lower envelopes much more easily. The resultant oscillatory part is not noticeably different whether this step is followed or not.
18 The wafers are grown with an identical MBE machine (VG V80H). The main reason the mobility of sample L is particularly low is that a poorly outgassed Si-dopant cell was used during its growth.
19 We attribute the larger scatter of data for Fig. 3(a) to the smaller oscillation amplitude (see the vertical axis); hence the worse signal-to-noise ratio.
21 The authors of Refs. 16 and 17 completely neglected the factor $A[\frac{1}{T_0(B)}]$. However, this does not affect their analysis much because the measurement temperature is low (1.5 K) in Ref. 16, and periods $a$ (at least for those experimental traces are shown) are rather large in Ref. 17.