

# Superconducting transition in wire network under spatially modulated magnetic field

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## Abstract

Superconducting wire network subjected to a uniform and a checkerboard-patterned magnetic field is investigated. The checkerboard field is created by an array of ferromagnetic dots placed on top of the superconducting network. The superconducting transition at  $\alpha = 1/2$  ( $\alpha$  being the frustration parameter) is studied in detail by measuring the current–voltage ( $I$ – $V$ ) characteristics. The vortex configuration in this case is doubly degenerate. We investigated the change of the transition according to lifting this degeneracy by applying the checkerboard field. The results are consistent with the scenario proposed by Korshnov [S.E. Korshnov, Phys. Rev. Lett. 88 (2002) 167007].

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The nature of phase transition in two-dimensional (2D) superconducting wire network (SWN) under magnetic field has been the subject of intensive studies both experimental and theoretical [1]. The relevant parameter on the physics of SWN under magnetic field is  $\alpha = \Phi/\Phi_0$ , called frustration parameter or vortex filling. It represents the average number of flux quanta per plaquette. Here,  $\Phi_0 = h/2e$  is the superconducting flux quantum and  $\Phi$  is the average external flux piercing the unit cell.

Upon cooling, the amplitude of the superconducting order parameter grows first around the mean field transition temperature  $T_{MF}$  of the superconducting material. When magnetic field is applied,  $T_{MF}$  shows periodic variations, known as the Little–Parks oscillations (LPO) [2,3]. The substructures within each period of LPO have an interesting one-to-one correspondence with the so-called Hofstadter butterfly [4,5], i.e., energy spectra of a 2D tight-binding model under a perpendicular magnetic field.

Just below  $T_{MF}$ , the amplitude of order parameter develops but the phase order remains undeveloped. Subsequent ordering of the phase over the entire system takes place at  $T_c < T_{MF}$ . This global phase ordering process is considered as a good experimental realization of the ordering problem of an XY spin system. In the absence of magnetic field ( $\alpha = 0$ ), this phase ordering transition is well-known as the so-called Kosterlitz–Thouless–Berezinskii (KTB) transition [1,6,7]. On the other hand, the nature of phase transition for non-zero  $\alpha$  is still not fully elucidated in spite of many theoretical and experimental studies [6–9].

In particular, the half-filled ( $\alpha = 1/2$ ) case, which is mapped to the so-called fully frustrated XY (FFXY) model, attracts much interest [10]. The characteristic feature of the FFXY model lies in the double degeneracy of ground state vortex configuration. This degeneracy results in the nucleation of Ising-type domain excitations (Fig. 1b), which destroy the global phase order of FFXY model as well as the vortex pair excitations (Fig. 1a) in usual (unfrustrated) XY model. In addition, as Korshnov has pointed out in Ref. [11], kink–antikink pair excitation (Fig. 1c)

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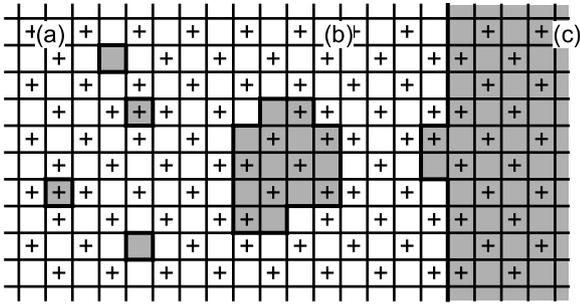


Fig. 1. Excitations in the FFXY model. (a) Vortex-antivortex pair, (b) antiphase domain, (c) kink-antikink pair on the domain wall.

plays important role in the transition of FFXY model because an antiphase domain becomes able to destroy the phase order only in the presence of free kinks on its boundary.

The nucleation and dissociation process of a kink pair is analogous to KTB transition for a vortex pair. When KTB transition temperature  $T_{\text{KTB}}$  is approached from below, phase stiffness is decreased by the destruction of the phase order due to the vortex pair excitation. KTB transition occurs when helicity modulus  $\Gamma$ , the quantity representing the phase stiffness, is decreased to  $\Gamma = (2/\pi)k_B T_{\text{KTB}}$ . Because  $\Gamma = 0$  for  $T_{\text{KTB}} < T$ ,  $\Gamma$  shows a jump to zero at KTB transition temperature, called universal jump. The kink transition temperature  $T_{\text{kink}}$  is determined from the relation  $\Gamma = (4/\pi)k_B T_{\text{kink}}$ , which leads to  $T_{\text{kink}} < T_{\text{KTB}}$ . Above  $T_{\text{kink}}$ , free kinks are nucleated and they effectively disturb the phase order across the domain boundary. It results in a sudden drop of  $\Gamma$ , so that vortex pair unbinding takes place concomitantly. At this “KTB-like” transition, the drop of  $\Gamma$  should be significantly larger than the universal jump for the canonical KTB transition.

Most of the experiments on SWN to date were conducted by use of uniform external magnetic field. Adding a spatially modulated magnetic field extends the parameter space. Using superconductor/ferromagnet hybrid structure, we apply spatially modulated magnetic field to SWN. We focus on the nature of superconducting transition in the fully frustrated case, and investigate the effect of the degeneracy-lifting field. More details of this work are published in Ref. [12].

The sample is an Al wire network with a regular array of ferromagnetic Co dots on top of the SWN. Fig. 2a is a scanning electron microscope image of the sample. Fig. 2b is a schematic crosssectional view, showing the Co dots overbridging the Al wire. The Al SWN and the Co dots were separated by a Ge layer, so that no pair-breaking effect was involved. When an in-plane magnetic field of sufficient strength (typically 500 mT) was applied as shown in Fig. 2b, the Co dots were fully magnetized and the stray field from the Co dots created a spatially modulated magnetic field. The Co dot arrangement shown in Fig. 2a generated a checkerboard flux pattern, i.e., alternating flux piercing the plaquettes of the SWN. The amplitude of the checkerboard-pattern flux, called  $\beta$  hereafter, is

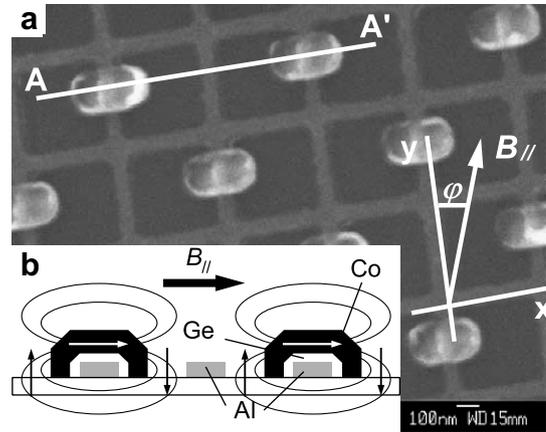


Fig. 2. (a) Scanning electron microscope image of the Al network decorated with an array of Co dots. The  $x$  and  $y$  axes and  $\varphi$  are indicated. (b) Schematic drawing of a cross section of A–A' indicated in (a).

defined in the same unit as  $\alpha$ , i.e., flux per plaquette in the unit of  $\Phi_0$ . If the magnetization of Co dots are saturated,  $\beta \propto \sin \varphi$ . We used a cross-coil magnet system, consisting of a horizontal split coil and a vertical solenoid, to independently control  $\alpha$  and  $\beta$ . The sample was mounted horizontal so that  $\alpha$  was controlled by the vertical field generated by the solenoid, while  $\beta$  was controlled by changing the azimuthal angle  $\varphi$  of the horizontal field by rotating the sample in the split coil magnet. Sample was directly immersed in superfluid helium and the temperature was stabilized to better than 2 mK by use of a suitable feedback circuitry.

First, we investigated the change of LPO due to the field modulation. We measured the resistance  $R$  of the SWN as a function of the perpendicular magnetic field  $B$  at different temperatures  $T$ . The results were then converted to  $T_{\text{MF}}(B)$ , which is defined as a temperature at which  $R(T, B) = 20 \Omega \approx 0.3R_n$ , where  $R_n$  is the normal state resistance. The measured LPO is shown in Fig. 3 by solid curves. The different curves in Fig. 3 corresponds to different settings of  $\varphi$  and hence different values of  $\beta$ . The dotted

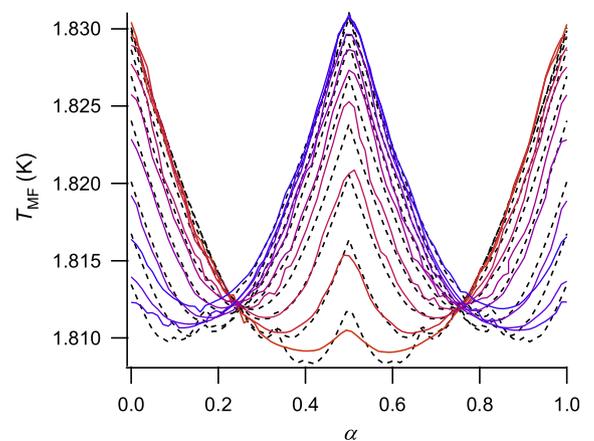


Fig. 3. Comparison of LPOs between experiment and calculation. Solid curves are experimental and dotted curves are calculated results.

curves show results of calculation from the spectral edge of corresponding Hofstadter butterfly [5]. The result shows good agreement and demonstrates that the checkerboard field with good uniformity and controllability is realized in this sample.

Measurement of the dc current–voltage ( $I$ – $V$ ) characteristics is a useful tool to probe into the nature of superconducting transition. If  $I$ – $V$  characteristics shows a power-law dependence  $V \propto I^a$ , power-law exponent  $a$  is directly related to the helicity modulus  $\Gamma$  as  $a = 1 + \pi\Gamma/k_B T$ . Therefore, from temperature dependence of  $a$  we can know how the global phase order is destructed with temperature increased.

First, we focus on the case with no staggered magnetic field ( $\beta = 0$ ). Solid circles in Fig. 4 shows the temperature dependence of the exponent  $a$  in the absence of the uniform and the modulated magnetic fields ( $\alpha = \beta = 0$ ). As temperature is lowered,  $a$  jumps from 1 to  $\sim 3$  at  $T \sim 1.79$  K and then increases gradually. This behavior is consistent with the KTB transition [1,7].

Triangles in Fig. 4 shows exponent  $a$  for the fully frustrated ( $\alpha = 1/2$ ) case. As compared with  $\alpha = 0$  case,  $T_c$  is suppressed and  $a$  increase steeper below  $T_c$ . These features are similar to those reported by van Wees et al. [6] for their Josephson junction array.

Next, we turn on the checkerboard magnetic field  $\beta$  and investigate its effect on the behavior for the fully frustrated case ( $\alpha = 1/2$ ). Fig. 5 shows the evolution of the temperature dependence of  $a$  for  $\alpha = 1/2$  with  $\beta$  changed from 0 to  $1/2$ . As  $\beta$  is increased from zero,  $T_c$  increases rapidly and is recovered to the value in the absence of magnetic field ( $\sim 1.8$  K) for  $1/4 < \beta < 1/2$ . Concomitantly, the temperature dependence of  $a$  below and near  $T_c$  becomes less steep. It should be noted that the data for  $\beta = 1/2$  are seemingly identical to those for the case of  $\alpha = \beta = 0$ . Namely, the canonical KTB transition is restored for this case.

As compared with the canonical KTB transition for  $\alpha = \beta = 0$ , the salient features of our data for fully frustrated ( $\alpha = 1/2$ ) case can be summarized as follows:

- (i) In the absence of the field modulation ( $\beta = 0$ ),
  - $T_c$  is suppressed.
  - The temperature dependence of  $a$  below and near  $T_c$  becomes steeper.

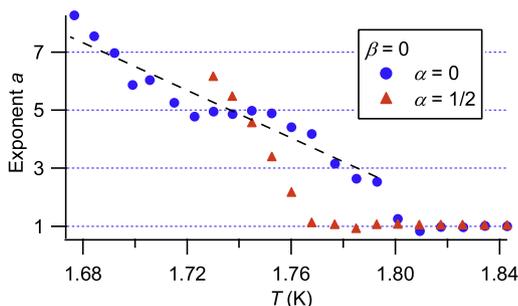


Fig. 4. Temperature dependence of the power-law exponent  $a$  in the absence of modulated field ( $\beta = 0$ ). Solid circles are the data for  $\alpha = 0$ . (The dashed line is guide to the eye). Triangles are the data for  $\alpha = 1/2$ .

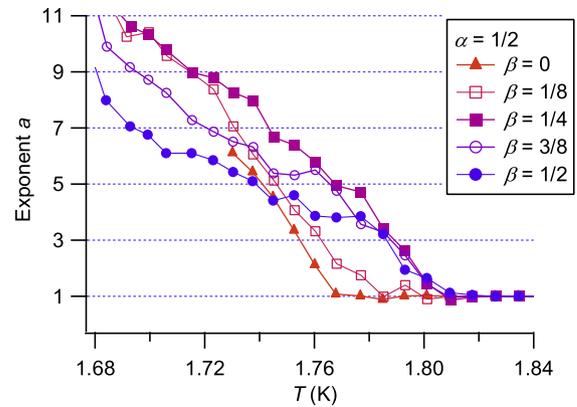


Fig. 5. Temperature dependence of the power-law exponent  $a$  of the  $I$ – $V$  characteristics at  $\alpha = 1/2$  for several values of  $\beta$ .

- (ii) As  $\beta$  is increased from 0 to  $1/4$ ,
  - The suppression of  $T_c$  is recovered.
  - The temperature dependence of  $a$  below and near  $T_c$  become less steep.
- (iii) In the range  $1/4 \leq \beta \leq 1/2$ ,
  - $T_c$  is not changed so much.
  - The temperature dependence of  $a$  below  $T_c$  becomes smaller as  $\beta$  is increased.
- (iv) At  $\beta = 1/2$ ,
  - The behavior of  $a$  is identical to that of KTB transition for  $\alpha = \beta = 0$  case.

The feature (i) agrees with the picture of “KTB-like” transition in FFX model. The feature (iv) is also easy to understand because in this case the degeneracy is lifted enough and only vortex pair excitation is allowed.

The features (ii) and (iii) refer to the effect of  $\beta$  on the phase transition at  $\alpha = 1/2$ . With increasing  $\beta$ , the antiphase domains become more and more costly, so that the antiphase domains will be smaller in size and distributed more sparsely. According to the scenario of KTB-like transition, the sudden drop of  $\Gamma$  at  $T_{\text{kink}}$  is because antiphase domains get able to destroy the phase order at the transition. Therefore smaller antiphase domains (larger  $\beta$ ) means smaller drop of  $\Gamma$ . As a result, the suppression of  $T_c$  and the reduction of  $a$  below  $T_c$  in feature (i) should be recovered for  $\beta$  is increased. This picture agrees with the feature (ii).

If  $\beta$  is large enough, the vortex pair is more effective on the phase disordering than the antiphase domain with free kinks. In this regime, the transition is solely governed by the vortex pair dissociation process, similar to canonical KTB transition. However, antiphase domains may affect the phase order below  $T_c$  even if they are irrelevant on the phase disordering process. In actuality, if a vortex comes close to an antiphase domain, it tends to merge with the domain. Therefore, only vortex pairs smaller than the mean distance between domains can survive, since larger ones are “absorbed” by the antiphase domains. As a result, the size of vortex pairs is smaller than the  $\beta = 1/2$  (KTB transition) case at the same temperature. It results in the recovering of  $a$  in comparison with  $\beta = 1/2$  case because

the larger vortex pairs are more effective on phase disordering. This picture at least qualitatively agrees with the experimental feature (iii).

In conclusion, the superconducting transition of SWN under uniform and modulated magnetic fields is studied by the measurement of  $I$ – $V$  characteristics. Transition at zero magnetic field ( $\alpha = \beta = 0$ ) is the KTB type. The fully frustrated case ( $\alpha = 1/2$  and  $\beta = 0$ ) exhibits a “KTB-like” transition with a drop of  $a$  larger than the universal jump, which is consistent with kink–antikink unbinding scenario. Application of a checkerboard field lifts the double degeneracy inherent to the system at  $\alpha = 1/2$ . The nature of transition at  $\alpha = 1/2$  is altered by increasing  $\beta$  from 0 to  $1/2$ , until the canonical KTB transition is restored at  $\beta = 1/2$ . In the range  $0 < \beta < 1/4$ , the kink–antikink unbinding scenario is still an appropriate description of the transition. For  $1/4 < \beta < 1/2$ , kinks are no longer effective in causing phase disorder. The behavior of  $a$  in this range can be qualitatively explained by considering vortex–antivortex pair excitation in the presence of sparse patches of antiphase domains.

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