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Superconducting Network in Spatially Modulated Magnetic Field — Hofstadter-Type Problem in Checkerboard Field

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The superconducting transition of a two-dimensional square network of Nb in a spatially modulated magnetic field, consisting of a uniform component and a sign-alternating (checkerboard pattern) component, has been studied. Evolution of the Little-Parks oscillation pattern as a function of the checkerboard field amplitude manifests some features of the low-energy envelopes of the corresponding Hofstadter butterfly-type diagrams.

KEYWORDS: superconducting network, Hofstadter butterfly, spatially modulated magnetic field

Two-dimensional superconducting wire networks and Josephson junction arrays have been studied extensively over the past two decades.[1] Developments of microfabrication techniques have made it possible to produce submicron-size networks and arrays of arbitrary shapes and symmetries. One of the most interesting aspects of these systems is the effect of a magnetic field, which introduces frustration associated with the phase of the superconducting wavefunction. A regular superconducting network subjected to a uniform external magnetic field provides a textbook example of a two-period system. The two competing periods in this problem are the unit cell size a and the magnetic length $\ell = (\hbar/2eB)^{1/2}$. The problem can be mapped onto the so-called Hofstadter problem of a two-dimensional Bloch band in magnetic field, which leads to the well-known butterfly diagram with intricate fractal structure.[2] The key parameter, $\alpha = Ba^2/\phi_0$, called the frustration index, is the average flux per unit cell in the unit of flux quantum $\phi_0 = h/e$ ($\phi_0 = h/2e$ for the case of superconducting networks). When the frustration index takes a rational value $\alpha = p/q$, p and q being mutually prime integers, the energy spectrum consists of q sets of bands.

If we consider the problem in terms of ordinary crystal lattices, the experimentally accessible values of the magnetic field can cover only a very limited range of α near zero. In contrast, the use of superconducting micronetworks allows us to access the entire range of α. For a square lattice of $1\,\mu$m unit cell, for example, the condition of one flux quantum per plaquette corresponds to $B = 2\,$mT. The types of superconducting network studied to date include regular square and hexagonal lattices, artificially disordered ones,[3] quasicrystalline ones,[4] and fractal ones.[5] To our knowledge, all reported experiments on superconducting networks have been performed with a uniform external magnetic field. In the present study, we explore a new physical situation by using a spatially varying magnetic field, which has been studied theoretically by several authors.[6-8] We investigate the response of a square lattice network to a spatially modulated external magnetic field which generates a checkerboard pattern of fluxes piercing the network. We compare the experimental result with the corresponding Hofstadter-type problem, i.e., square lattice in a checkerboard pattern magnetic field, which is theoretically examined in an accompanying paper.[9]

A scanning electron micrograph and an illustration of the sample used in the present study are shown in Fig. 1. The sample was fabricated on a SiO_2/Si(100) substrate using electron beam lithography, ion beam sputtering deposition and the lift-off process. The network consisted of 80×80 square unit cells of size $0.5\,\mu\text{m} \times 0.5\,\mu$m com-

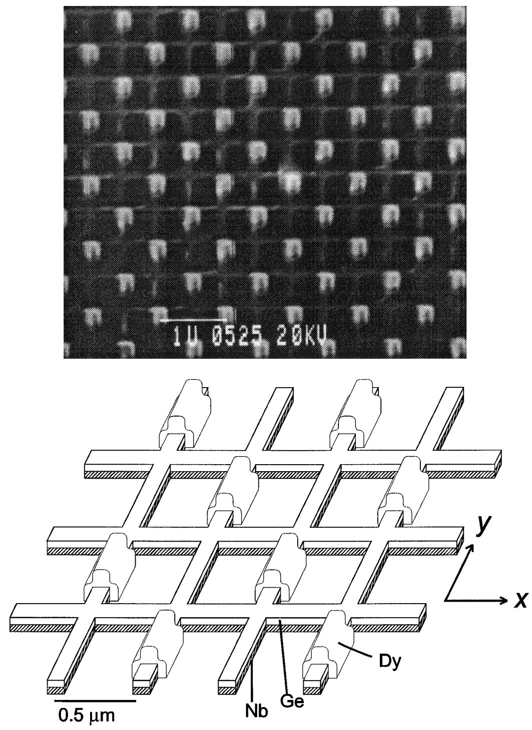

Fig. 1. Scanning electron micrograph of the Nb square network with Dy islands. The drawing is a schematic of the sample structure. The x and y axes are defined as shown.

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posed of Nb wires 40 nm thick and 100 nm wide. The surface of the Nb network was covered with a 40 nm-thick Ge layer, which served as a spacer, preventing the direct contact of Nb with the ferromagnetic material to be deposited in the second fabrication step. In the second fabrication step, small islands of Dy were formed atop every second bond in the y-direction, as depicted in Fig. 1. The Dy islands were $200\,\mathrm{nm} \times 250\,\mathrm{nm}$ rectangles with thickness of 100 nm.

The measurements of the superconducting properties were performed in a cross-coil superconducting magnet system equipped with a variable temperature insert. The cross-coil system consisted of a split-coil magnet which generates a horizontal field up to 6 T, and a small solenoid which generates a vertical field up to 1 T. The superconducting transition temperature of the Nb network at zero magnetic field was $T_c = 6.5\,\mathrm{K}$. Using a feedback circuit, the temperature was controlled in the vicinity of T_c with stablity better than $2\,\mathrm{mK}$. The sample was set with the plane aligned horizontally, within a few degrees. The vertical-field solenoid produced a uniform magnetic field B_{perp} perpendicular to the network plane. The horizontal field $B_\parallel$ was used to control the magnetization of the Dy islands. The Dy islands showed little hysteresis and it was found that the magnetization was approximately proportional to the applied field in the range of field used in the present study.

When $B_\parallel$ is applied along the x-axis, the perpendicular component of the fringing field generated by the array of Dy islands produces a sign-alternating (checkerboard) field pattern on the square network. The amplitude of the sign-alternating component increases approximately linearly with $B_\parallel$. On the other hand, when $B_\parallel$ is parallel to the y-axis, the perpendicular component of the fringing field pierces each plaquette equally, so it results in no spatially varying component. In both cases, due to the nonzero misorientation of the sample, $B_\parallel$ adds a small background field $B_\parallel \sin\delta$ to $B_\perp$, δ being the misorientation angle which is typically a few degree.

We investigated the resistive transition of the network as a function of the uniform field and the checkerboard field. The resistivity measurement was performed by a standard four-wire *ac* method, with a typical excitation current of $1\,\mu\mathrm{A}$. The measurement procedure was as follows: The temperature was stabilized at a point slightly below T_c and near the low resisitivity end of resistive transition. The resistance of the network was measured as a function of $B_\perp$. The measurement was repeated with incremental changes of $B_\parallel$. Because of a weak but non-zero dependence of T_c on $B_\parallel$, a minute adjustment of temperature was necessary to stay at the same position in the resistive transition each time $B_\parallel$ was changed.

Figure 2 shows magnetoresistance traces for different values of $B_\parallel$ applied along the x-direction. Each trace is vertically shifted for visual clarity. The magnetoresistance $R(B_\perp)$ exhibits an oscillatory pattern which reflects the Little-Parks oscillation of T_c with $B_\perp$. Small dips near the resistance maxima correspond to local minima due to a stable fluxoid configuration at $\alpha = 1/2$ (modulo 1) (i.e., checkerboard pattern arrangement of fluxoids). As $B_\parallel$ is increased, these dips become deeper

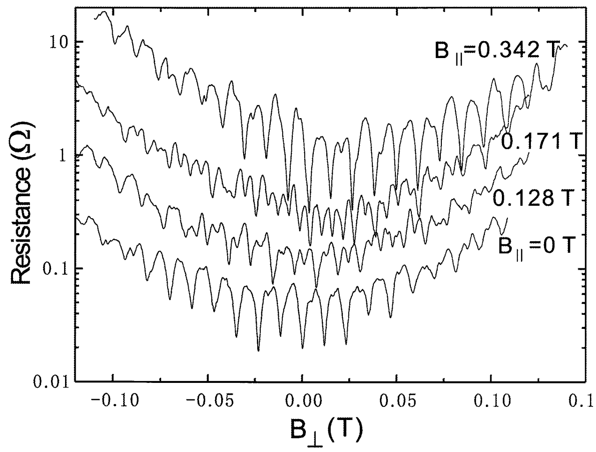

Fig. 2. Resistance of the network as a function of the perpendicular field $B_\perp$ for different values of the parallel field $B_\parallel$. Each trace is shifted vertically for visual clarity.

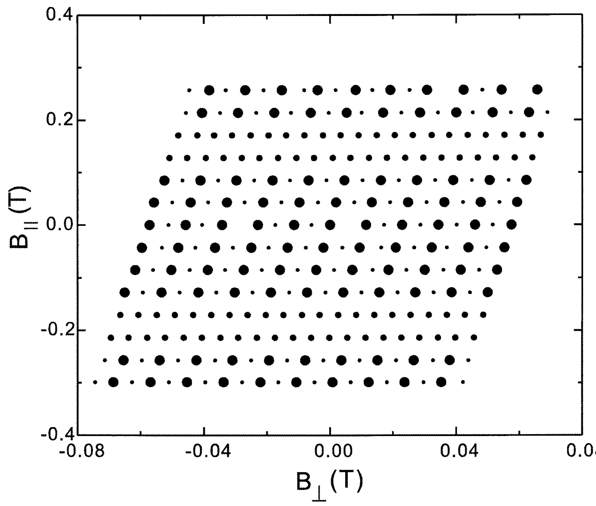

Fig. 3. Map of the positions of the resistance minima in the $(B_\perp, B_\parallel)$-plane. The size of each dot schematically represents the strength of the dip feature, i.e., the relative depth of the resistance minima.

until they become comparable to those at integer α when $B_\parallel = 0.171\,\mathrm{T}$. When the horizontal field is increased to $B_\parallel = 0.342\,\mathrm{T}$, the magnetoresistance pattern becomes similar to the one at $B_\parallel = 0$. This change of magnetoresistance pattern with $B_\parallel$ repeats itself with a period $\Delta B_\parallel \sim 0.34\,\mathrm{T}$. Similar measurements with $B_\parallel$ applied along the y-direction revealed no such change in the Little-Parks oscillation pattern, as expected.

To see the evolution of the oscillatory pattern more clearly, we map the positions of the resistance minima on a $(B_\perp, B_\parallel)$-plane in Fig. 3. Each trace such as that shown in Fig. 2 gives a horizontal row of dots. The size of each dot in this plot is intended to represent the strength of the feature, i.e., the relative depth of the dip structure, in a qualitative sense. The slanting of this dot map is due to a small misorientation of the $B_\parallel$ field away from the direction exactly parallel to the plane of the network, which results in a nonzero perpendicular component. For the particular set of data shown in Fig. 3, the misorientation angle is ~ 3.0 degrees. At $B_\parallel = 0$, the large dots correspond to the large dips at integer values of α and the small ones to the features at $\alpha = 1/2$ (modulo 1). As $B_{\parallel}$ is varied, the small dots evolve to large ones and vice versa. The trace at $B_{\parallel} = 0.171\,\mathrm{T}$ in Fig. 2 corresponds to the midpoint of this evolution. From this plot, it can be seen that although the oscillatory pattern at $B_{\parallel} = 0.342\,\mathrm{T}$ in Fig. 2 is similar to the one at $B_{\parallel} = 0$, the large dips correspond to $\alpha = 1/2$ (modulo 1) and the small dips to integer α. Thus, the genuine periodicity in $B_{\parallel}$ is $2 \times 0.34 = 0.68\,\mathrm{T}$.

Let us now consider the corresponding Hofstadter-type problem, i.e., a square network in a checkerboard pattern magnetic field. The checkerboard field can be decomposed into a uniform component and a sign-alternating component. The effect of these fields can be represented by assigning suitable Peierls phase factors to each bond of the square lattice, as shown in Fig. 4. Here, α and β are the parameters representing the phase factors associated with the uniform and the sign-alternating component, respectively. The Schrödinger equation for a wavefunction $\Psi(n, m)$, where (n, m) is the index of lattice sites, can be written as[10)]

$$\begin{aligned}\epsilon(\alpha,\beta)\Psi(n,m) = {} & \Psi(n-1,m) + \Psi(n+1,m) \\ & + \mathrm{e}^{\mathrm{i}(-1)^{n+m}\pi\beta}\big[\mathrm{e}^{\mathrm{i}2\pi n\alpha}\Psi(n,m-1) \\ & + \mathrm{e}^{-\mathrm{i}2\pi n\alpha}\Psi(n,m+1)\big], \end{aligned} \tag{1}$$

The above equation differs from the one for the original Hofstadter problem[2)] by an extra phase factor $\mathrm{e}^{i(-1)^{n+m}\pi\beta}$ which arises from the sign-alternating field component. The energy spectra derived from the above equation are discussed in the accompanying paper.[9)] In that paper, the energy spectra as a function of α (butterfly diagrams) were calculated for different values of β. The low-energy envelope of each butterfly diagram gives the α-dependence of the ground-state energy for the particular value of β.

Figure 4 shows a three-dimensional plot of the ground-state energy as a function of α and β. The α-dependence curve at integer values of β represents the standard pattern of a single cycle of the Littel-Parks oscillation in a superconducting square network subjected to a uniform magnetic field. It has fine structures with local minima at $\alpha = 1/2, 1/3, 1/4, 2/5, \cdots$ correponding to the commensurate flux configurations. The β-dependence, by comparison, is monotonic and simple. As evident from the top drawing in Fig. 4, the Peierls phase pattern representing the effect of the checkerboard flux repeats itself with the period of unit cell consisting of two praquettes. Therefore, unlike the effect of $\alpha = p/q$ which involves a large unit cell when the denominator is a large number, the effect of β for its arbitrary value can be treated analytically.

Here, we focus on the ground-state energies at $\alpha = 0$ and $1/2$ (modulo 1), which can be obtained simply as

$$\begin{aligned}\epsilon(\alpha = 0, \beta) &= -4\cos\Big(\frac{\pi}{2}\beta\Big) \\ \epsilon\Big(\alpha = \frac{1}{2}, \beta\Big) &= -4\cos\Big(\frac{\pi}{2}\Big(\frac{1}{2} - \beta\Big)\Big). \end{aligned} \tag{2}$$

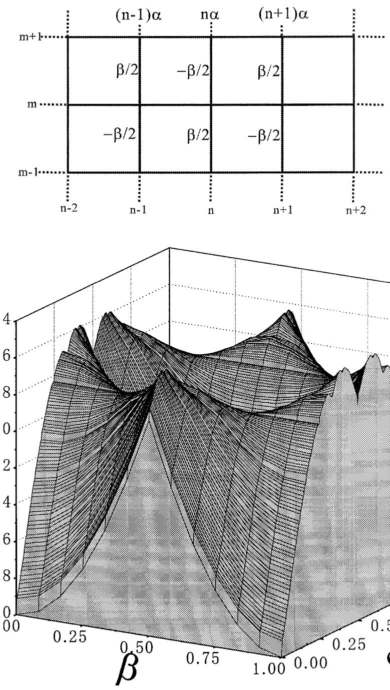

Fig. 4. The top drawing shows the model of the square network under a checkerboard magnetic field. The relevant Peierls phase factors are assigned to each bond. The lower figure shows the dependence of the ground-state energy of the square network on the parameters α (uniform component) and β (sign-alternating component).

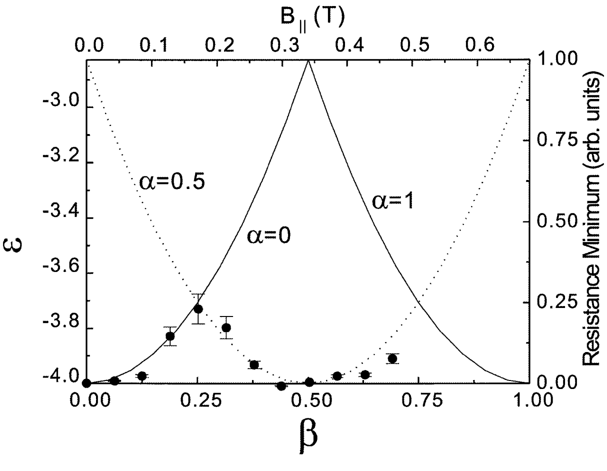

Fig. 5. The solid and dotted curves represent the β-dependence of the ground-state energy at $\alpha = 0, 1$ and $\alpha = 1/2$, respectively. Crossover of the global ground state from $\alpha = 0$ to $1/2$ occurs at $\beta = 1/4$ The solid circles (right and top axes) represent the relative values of the resistance minima.

The curves in Fig. 5 show the dependence of these quantities on β. Throughout the relevant range they are quadratic in β. As β is increased from zero, a crossover of the global ground state from $\alpha = 0$ to $\alpha = 1/2$ occurs at $\beta = 1/4$. This behavior is corroborated by the data shown in Figs. 2 and 3. The solid circles in Fig. 5 represent the change in the relative depth of the resistance dip with $B_{\parallel}$, scaled to obtain a best-fit with the $\epsilon(\alpha, \beta)$ curves. Two things should be noted about this comparison. First, the resistance change reflects and mimics the T_c change, but they are not exactly the same. For a quantitative comparison, the detailed shape of the resistive transition has to be considered. Second, the experimental data contain substantial uncertainty arising from period-to-period fluctuations in the magnetoresistance pattern. Nevertheless, it is evident that the evolution of the resistance dips with the sign-alternating field component reproduces the β-dependence of the global ground-state energy, at least semiquantitatively.

The origin of the period-to-period fluctuations in the oscillation waveform is not clear at present. One of the factors to be considered in detail is the change in the magnetization of the Dy islands when $B_\perp$ is swept. The change in the orientation of the total external field affects the magnetization of the Dy islands and may change the fringing field pattern in subtle ways. Another factor of possible relevance is the penetration of fluxoids into the Nb wires, whose width is comparable to the Ginzburg-Landau coherence length in the temperature range of the experiment. Experiments using samples with different materials and sizes of the superconducting network and the ferromegnetic islands are in progress to clarify this point.

In summary, we have investigated the resistive transition of a two-dimensional square network of Nb under the influence of a checkerboard pattern of magnetic field. The evolution of the Little-Parks oscillation pattern as a function of the amplitude of the sign-alternating field component manifests some features of the low-energy envelopes of the corresponding Hofstadter butterfly-type diagrams.

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