SUPERCONDUCTING NETWORK WITH MAGNETIC DECORATION — HOFSTADTER BUTTERFLY IN SPATIALLY MODULATED MAGNETIC FIELD —

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We calculate the Hofstadter butterfly diagram for a two-dimensional tight binding square lattice subjected to spatially varying flux patterns. Samples of superconducting wire network decorated with mesoscopic ferromagnet array exhibit characteristic Little-Parks oscillation patterns reminiscent of the edge shape of the corresponding Hofstadter spectra.

1. Introduction

Energy spectrum of tight-binding electrons on a two-dimensional (2D) square lattice under a magnetic field perpendicular to the plane has an exquisite structure, known as Hofstadter butterfly.[1,2] Spectra for 2D lattices with a variety of symmetry have been calculated.[3–6] Experimental verification of the full Hofstadter butterfly spectra is a formidable task, although such an attempt using semiconductor lateral superlattice seems to attain a certain degree of success.[7] Superconducting wire network constitutes an experimental system intimately related to the Hofstadter problem. It has been demonstrated that the Little-Parks oscillation of transition temperature T_c of a superconducting wire network exhibits fine structures at commensurate values of average flux per unit cell, and that $\Delta T_c(H)$ reproduces the edge shape of the Hofstadter spectrum for the corresponding 2D lattice.[8]

These and the majority of subsequent studies were conducted in a uniform magnetic field. In the present study, we address ourselves to the case of spatially varying magnetic field, extending our previous work.[9] In the next Section, we calculate the spectra for a square lattice subjected to both a spatially varying and a uniform magnetic field. Section 3 describes experiment using superconductor/ferromagnet hybrid systems. A brief summary is given in Section 4.

2. Hofstadter spectra for spatially varying magnetic field

2.1. *Hofstadter problem*

Among different methods to calculate the Hofstadter spectrum, we adopt matrix diagonalization method, which is suitable for introduction of spatially varying magnetic field. We consider a 2D square lattice with lattice constant a and nearest neighbor transfer integral t. The Schrödinger equation for this system under a uniform magnetic field H reads

$$\varepsilon\psi_{n,m} = t\big[\psi_{n-1,m} + \psi_{n+1,m} + e^{-2\pi in\alpha}\psi_{n,m-1} + e^{2\pi in\alpha}\psi_{n,m+1}\big]. \tag{1}$$

with appropriate choice of the gauge and hence the Peierls phase factor. Here, $\psi_{n,m}$ represents the wave function at the lattice site (n,m), and $\alpha \equiv \phi/\phi_0 = eHa^2/h$ is the magnetic flux per plaquette measured in the unit of the flux quantum $\phi_0 = e/h$. The above equation can be reduced, by putting $\psi_{n,m} = e^{ik_x na}e^{ik_y ma}\varphi_n$, to the following one-dimensional Harper equation.

$$\varepsilon\varphi_n = t\big[e^{-ik_x a}\varphi_{n-1} + e^{ik_x a}\varphi_{n+1} + 2\cos(k_y a - 2\pi n\alpha)\varphi_n\big]. \tag{2}$$

When α is a rational number p/q, p and q being mutually prime, the size of the magnetic unit cell becomes $[qa, a]$, namely, $\psi(x+qa, y) = \psi(x,y)$ and $\psi(x, y+a) = \psi(x,y)$. The eigenvalues obtained by solving the secular equation constitute the well-known butterfly spectrum, which is reproduced on the topmost panel of Fig. 1.

2.2. *Checkerboard field*

We next consider the case of spatially varying magnetic field. The checkerboard field pattern (flux configuration alternating in sign on adjacent cells) is shown on the left panel of Fig. 2. The parameter β denotes the amplitude of the alternating field in the unit of quantum flux per unit cell. The assignment of the Peiels phase to each bond is represented by the number of arrowheads. The magnetic unit cell for $\alpha = p/q$ is $[2qa, a]$, *i.e.* the system is invariant under translation by $(2qa, 0)$ or $(0, a)$. The relevant equation reads

$$\begin{aligned}
\varepsilon\psi_{n,m} &= t\big[\psi_{n-1,m} + \psi_{n+1,m} + e^{-2\pi i(n-m+1)\alpha}\psi_{n,m-1} + e^{2\pi i(n-m)\alpha+2\pi i\beta}\psi_{n,m+1}\big] \\
&\qquad\qquad\qquad\qquad\qquad\qquad (n-m\text{: odd}) \\
\varepsilon\psi_{n,m} &= t\big[\psi_{n-1,m} + \psi_{n+1,m} + e^{-2\pi i(n-m+1)\alpha+2\pi i\beta}\psi_{n,m-1} + e^{2\pi i(n-m)\alpha}\psi_{n,m+1}\big] \\
&\qquad\qquad\qquad\qquad\qquad\qquad (n-m\text{: even})
\end{aligned} \tag{3}$$

The eigenvalue spectra for different values of β are shown on the left side of Fig. 1. They are symmetric with respect to transformations, $\beta \to 1 \pm \beta$. The spectrum for $\beta = 1/2$ is identical with that for $\beta = 0$ except shifted by $1/2$ along the α axis.

2.3. *Stripe field*

The Peierls phase assignment for the case of stripe field (one-dimensional modulation alternating in sign for adjacent columns) is given on the right panel of Fig. 2. Thre relevant equation is written as

$$\begin{aligned}
\varepsilon\psi_{n,m} &= t\big[\psi_{n-1,m} + \psi_{n+1,m} + e^{-2\pi in\alpha+2\pi i\beta'}\psi_{n,m-1} + e^{2\pi in\alpha+2\pi i\beta'}\psi_{n,m+1}\big) \quad (n\text{: odd}) \\
\varepsilon\psi_{n,m} &= t\big(\psi_{n-1,m} + \psi_{n+1,m} + e^{-2\pi in\alpha}\psi_{n,m-1} + e^{2\pi in\alpha}\psi_{n,m+1}\big] \qquad (n\text{: even})
\end{aligned} \tag{4}$$

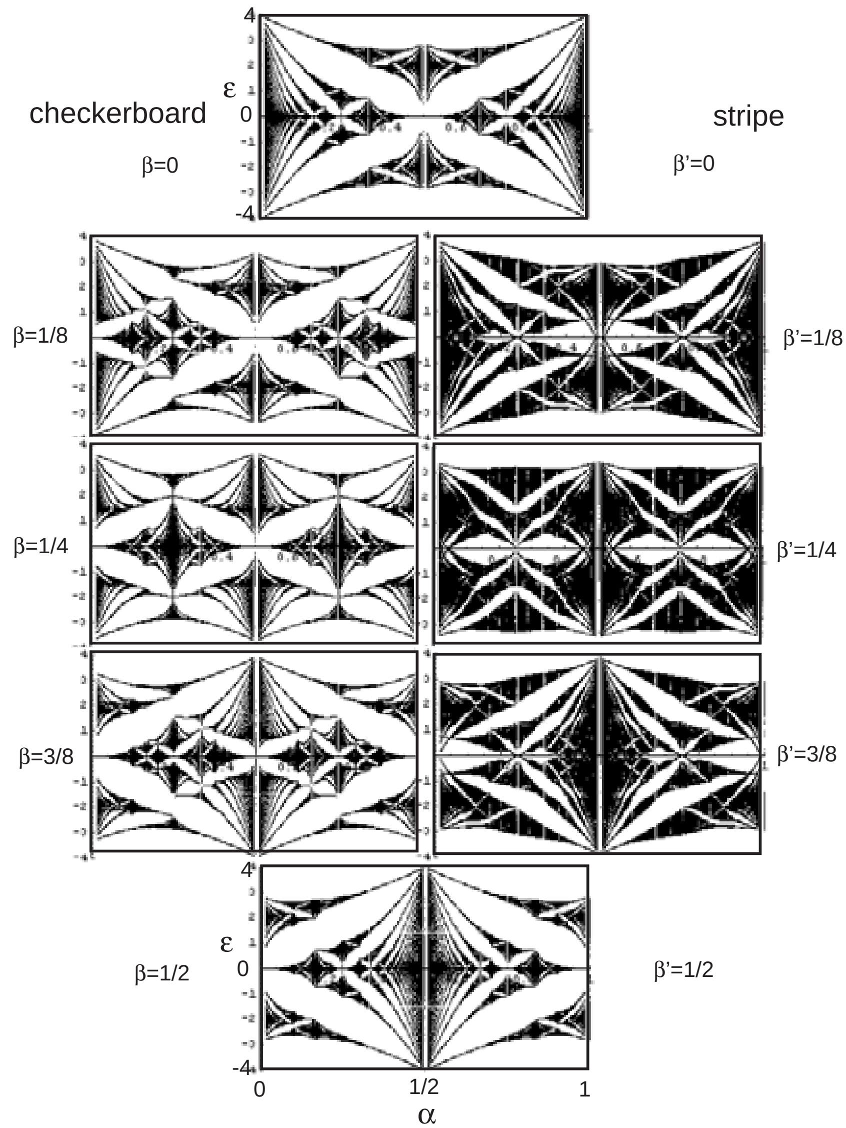

Figure 1. The top panel is the original Hofstadter spectrum for the uniform magnetic field case. The panels on the left side are the spectra under a spatially varying field of checkerboard patten with the modulation amplitude represented by the parameter β. Those on the right side are for the stripe field case (β'). The spectra for $\beta = 1/2$ and $\beta' = 1/2$ are identical and shown on the bottom panel, which is none other than the one on the top panel shifted by a half period.

The eigenvalue spectra are given on the right side of Fig. 1. The spectra for $\beta' = 0$ and $1/2$ are identical with those for the checkerboard field case. As evident from comparison between the two panels for $\beta = 1/8$ and $\beta' = 1/8$, introduction of a non-zero checkerboard field gives rise to spliting of the band center at $\alpha = 0$,

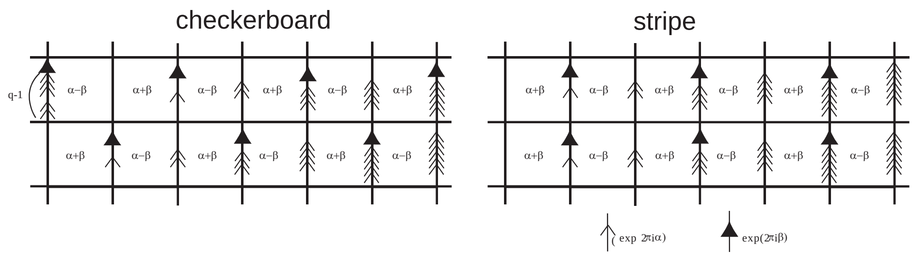

Figure 2. Assignment of Peierls phase factors to each bond of a square lattice, for checkerboard field (*left*) and stripe field (*right*).

while conspicuous splitting occurs at $\alpha = 1/2$ for the stripe field. It is also clear that the spectra for stripe fields are generally richer in black part than those for checkerboard fields. This is in line with a general trend that Hofstadter spectrum becomes blacker (more bands, less gaps) as one introduces anisotropy in the square lattice[5].

3. Magnetically decorated superconducting wire network

Now we turn to experiments using superconducting wire network. The change in the transition temperature ΔT_c of a square network of superconducting wire is expressed as

$$\frac{\Delta T_c(H)}{T_{c0}} = \frac{\xi^2(0)}{a^2} \arccos^2\left(\frac{\varepsilon_{\max}}{4}\right), \tag{5}$$

where $\varepsilon_{\max}$ represents the maximum eigenvalue for a given value of α [a], and $\xi(0)$ is the superconducting coherence length at zero temperature. Thus, the fine structure of Little-Parks oscillation reproduces the edge of the Hofstadter butterfly spectrum.

We fabricated superconductor/ferromagnet hybrid systems depicted in Fig. 3. A square network of Nb wire (lattice period 500 nm) was covered by a Ge layer for insulation, and an array of ferromagnetic substance (either checkerboard or stripe pattern) was deposite on top as shown in the figure. The magnetization of the ferromagnetic array was fixed at the saturation value by applying an external field ($\sim$ 0.5 T) parallel to the film plane. The amplitude of the spatially alternating field (parameter β or β') was controlled by changing the azimuthal angle of the parallel field. The resistance of the superconducting network was measured as a function of perpendicular field (parameter α) while keeping the sample at a constant temperature slightly below T_c.

Figure 3 shows the traces of magnetoresistance as a function of the perpendicular field (α). The evolution of the Little-Parks oscillation with incremental change of

[a]Note the difference in the definition of α by a factor 2 between the superconducting and normal systems associated with the difference in the definition of flux quantum ϕ_0.

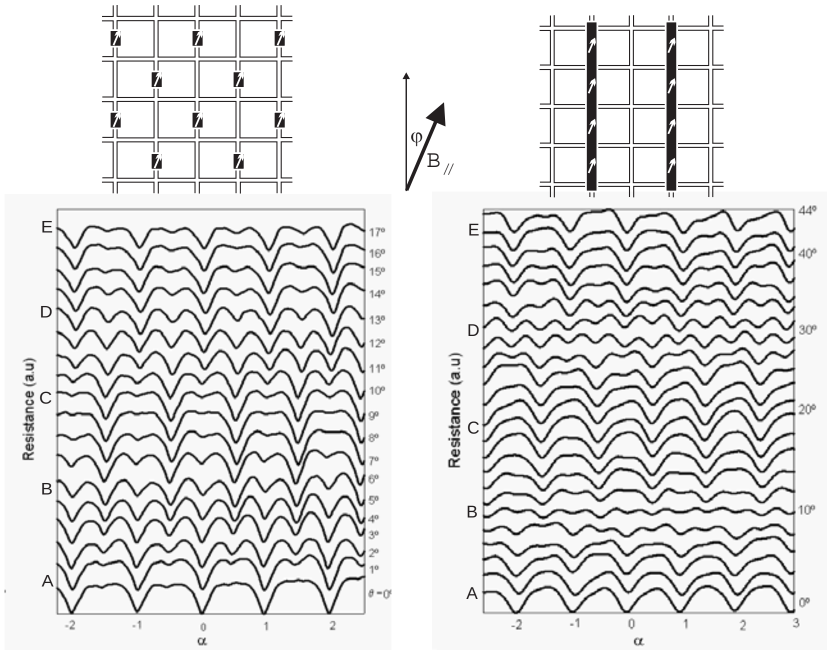

Figure 3. The Little-Parks oscillation of the magnetically decorated superconduting wire network samples. The structures of the superconductor/ferromagnet hybrid systems are shown at the top.

the parameter β (or β') by rotation of the parallel field is shown in the figure. Let us first look at the checkerboard field case, shown on the left panel. The trace marked A corresponds to $\beta = 0$. With increasing β, the dips at half-integer values of α deepen and those at integer values shallow, until they become equal for the trace B. Here, the period of oscillation becomes half the original one. This corresponds to $\beta = 1/4$. In the trace C, which corresponds to $\beta = 1/2$, the roles of integer and half-integer are interchanged, or in other words, the trace is shifted by a half period from the original one (trace A). The evolution with β thus goes on until it completes a full cycle at the trace E.

The behavior in the stripe field case on the right panel is similar, but differs in a few respects. Firstly, the oscillation amplitude at $\beta' = 1/4$ (trace B) is much smaller than the checkerboard field case. Secondly, shallowing of the dips at integer α as β' is increased from zero occurs more rapidly than the checkerboard field case. These features are in good agreement with the behavior of the edge of the Hofstadter spectra, as demonstrated in Fig. 4 which shows the evolution of the dip depth at $\alpha = 0$ with the value of β or β', in comparison with the edge of the corresponding Hofstadter butterfly spectra.

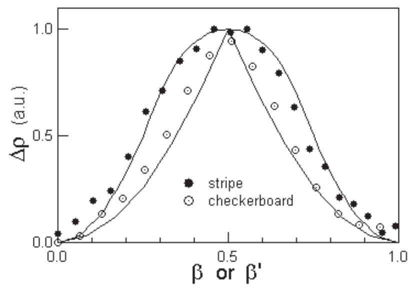

Figure 4. The symbols are the experimental data showing the relative change in the resistance at $\alpha = 0$ with β or β'. The curves represent the bottom edge of the corresponding Hofstadter spectra.

4. Conlusion

In conclusion, we have calculated the Hofstadter spectra for 2D square lattice subjected to a spatially varying magnetic field with different symmetries. We have realized the corresponding experimental systems by use of magnetically decorated superconducting wire network samples, and have demonstrated that their Little-Parks oscillation reflects the edge of the correpsonding Hofstadter spectra.

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