Spin Transport

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According to DARPA's definition, "Spintronics"[1] means "Spin Transport Electronics". The issue "Spin Transport" thus covers all the fields of spintronics and one-hour lecture for that is impossible. Instead, I would like to give a lecture on a few selected topics on "Spin Dependent Electron Transport in Bulk and Nano-structures". Because of the time limitation, I cannot go into the details of calculation, and I would like to give a very basic to consider the transport and peculiar transport phenomena in spin-polarized systems like ferromangets.

1 Introduction

Remember the Drude conductivity[2]

$$\sigma = \frac{ne^2\tau}{m}. \tag{1}$$

It is easy to see the conductivity is dominated by the number of carriers n (*i.e.*, the density of states), the effective mass m and the scattering time τ. In semiconductor nanostructures, we can modify or design all of these three parameters to obtain desired functions in electronic devices. In semiconductor spintronics devices, we therefore utilize the differences in n, τ, m for up and down spins to control spin-current (and charge current).

In metals, on the other hand, the main tunable parameter is τ and anomalous transport in spin-polarized metals mainly occurs through anomalies in τ. In hybrid materials such as metallic superlattices, however, the difference in the density of states can also be utilized as in giant magnetoresistance (GMR) and tunneling magnetoresistance (TMR) devices[3].

In this lecture, let us forget the effect of mass and consider how τ and the density of states $D(\epsilon)$ affect the transport. In the next section, we first review the phenomenological treatment of electric resistivity in materials with magnetic order. Then we go into various anomalous transport due to spin-dependent impurity scattering[4].

2 Spin Scattering Resistivity

In this section we consider the resistivity due to spin-dependent scattering in bulk systems. An important issue we pass here is the Kondo effect, which we will see in detail in a quantum dot system.

2.1 Conduction in Magnetic Metals —- Phenomenology

Here we only consider single-domain problem. The electric field vector $\boldsymbol{E}$ and the current vector $\boldsymbol{J}$ are related with the resistivity tesor $\boldsymbol{\rho}$ as

$$E_i = \sum_j \rho_{ij} J_j. \tag{2}$$

For simplicity, let the system be isotropic. Then the resistivity can be written as

$$\boldsymbol{\rho} = \begin{pmatrix} \rho_\perp(B) & -\rho_{\mathrm{H}}(B) & 0 \\ \rho_{\mathrm{H}}(B) & \rho_\perp(B) & 0 \\ 0 & 0 & \rho_\parallel(B) \end{pmatrix}, \tag{3}$$

and combination with (2) results in the expression

$$\boldsymbol{E} = \rho_\perp(B)\boldsymbol{J} + [\rho_\parallel(B) - \rho_\perp(B)][\boldsymbol{\alpha}\cdot\boldsymbol{J}]\boldsymbol{\alpha} + \rho_{\mathrm{H}}(B)\boldsymbol{\alpha}\times\boldsymbol{J}, \tag{4}$$

where $\boldsymbol{\alpha}$ is the unit vector with the direction of the magnetization [5]. And the flux density $\boldsymbol{B}$ is written with the demagnetization coefficient D, the magnetization $\boldsymbol{M}$ and the external field $\boldsymbol{H}$ as

$$\boldsymbol{B} = \boldsymbol{H} + 2\pi\boldsymbol{M}(1-D). \tag{5}$$

Conventionally ρ_{ij} can be devided into $\rho^0_{ij}(B)$ which goes to zero at $B=0$ and the remaining part without magnetic response as

$$\rho_{ij} = \rho^0_{ij}(B) + \rho^a_{ij}. \tag{6}$$

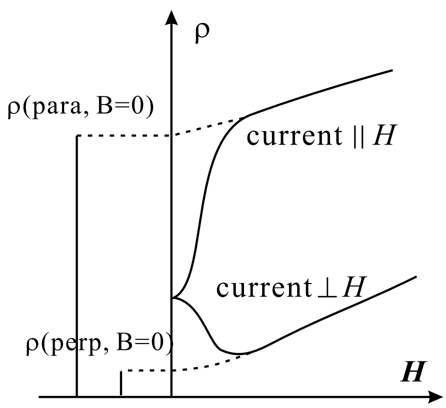

Figure 1: Schematically shows anisotropic magnetoressitance in an isotropic magnetic material.

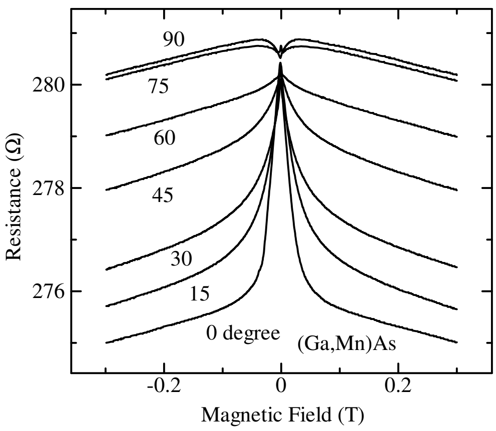

Figure 2: An example of anisotropic magnetoresistance. The material is (Ga,Mn)As (Mn content 5%) 200nm thin film. The growth surface is (001) and the current direction is [110]. T_c =60K, temperature of the measurement is 20K.

In a real magnet, the resistance around the zero-field is complicated and shows hysteretic behavior. Hence the separation in (6) is, in strict description, impossible. However as shown in Fig.1, if we can defince the division (6) from extrapolation to $B = 0$. Now $\rho^a_\parallel$, $\rho^a_\perp$ and $\rho^a_{\rm H}$ have the following meanings.

- $\rho^a_\parallel$: resistivity for current parallel to the magnetization
- $\rho^a_\perp$: resistivity for current perpendicular to the magnetization
- $\rho^a_{\rm H}$: anomalous Hall resistivity

Now from

$$\boldsymbol{\rho} = \boldsymbol{E}\cdot\boldsymbol{J}/|\boldsymbol{J}|^2$$

the resistivity is written as

$$\rho(0) = \frac{\rho_\parallel + 2\rho_\perp}{3} + \left(\cos^2\theta - \frac{1}{3}\right)(\rho_\parallel - \rho_\perp). \quad (7)$$

This equation expresses phenomenological understanding of the anisotropic magnetoresistance (AMR) effect schematically shown in Fig.1. The amplitude is

$$\mathit{\Delta}\rho/\rho = (\rho_\parallel - \rho_\perp)/(\frac{1}{3}\rho_\parallel + \frac{2}{3}\rho_\perp), \quad (8)$$

and it is also understood that the sign may change with the order of $\rho_\parallel$, $\rho_\perp$.

The non-diagonal terms represent the anomalous electric field and coefficient as

$$\boldsymbol{E}_{\rm H} = \rho_{\rm H}\boldsymbol{\alpha}\times\boldsymbol{J}, \quad R_{\rm aH} = \rho_{\rm H}/4\pi M, \quad (9)$$

while normal Hall electric field is expressed as

$$E_y = (\rho_\parallel - \rho_\perp)\cos\theta\sin\theta J, \quad (10)$$

where θ is the angle between $\boldsymbol{J}$ and $\boldsymbol{M}$.

Figure 2 shows an example of AMR in a diluted magnetic semiconductor (Ga,Mn)As.

2.2 Critical Scattering at T_c

Now we go into a scattering problem [6]. In a simplest approximation the resistivity ρ is the sum of that by impurity scattering and by phonon scattering.

$$\rho(T) = \rho_0 + \rho_{\rm P}(T). \quad (11)$$

ρ_0 may also depend on temperature especially for semiconductors. In semiconductors, because of low electron density the average de Broglie wavelength λ of carriers varies with temperature. When λ crosses the average distance of impurities, strong backscattering effect due to the quantum interference occurs resulting in a hump structure in the temperature dependence of resistivity[7].

A similar phenomenon occurs around the critical temperature T_c of a ferromagnet. This is, in a crude way, explained as follows. We first consider the case far above T_c, so the local moments can be treated as free paramagnetic impurities. Between a conduction electron spin $\boldsymbol{s}$ and a local moment $\boldsymbol{S}$ at the origin, there exist so called exchange interactions with various origins, which can be summarized as

$$J_{\rm H}\boldsymbol{s}\cdot\boldsymbol{S}|\psi_s(0)|^2\mathit{\Omega} \equiv \mathit{\Gamma}\boldsymbol{s}\cdot\boldsymbol{S}, \quad (12)$$

where $J_{\rm H}$ is (generalized) Hund coupling energy, ψ_s is the wavefunction of the conduction electron and $\mathit{\Omega}$ is the atomic volume. This type of effective Hamiltonian appears in many cases, one of which we will see in the next section for a quantum dot system.

Starting from the interaction (12), the temperature independent resistivity $\rho_{\rm M}$ due to the magnetic impurity scattering is calculated by the linear response theory as (Appendix A)

$$\rho_{\rm M} = \frac{k_{\rm F}(m\mathit{\Gamma})^2}{4\pi e^2 z\hbar^3}S(S+1), \quad (13)$$

where z is the number of electrons per one impurity site. Here the S^2 term comes from non spin-flip scattering (interaction s_zS_z) and the S term comes form spin-flip scattering (interaction $s^-S^+ + s^+S^-$).

Below T_c in a three dimensional system, a long range order appears. The scattering is due to the fluctuation from the ordered state and the resultant resisitivity $\rho_{\rm m}$ is expressed as

$$\rho_{\rm m} = \frac{\rho_{\rm M}}{S(S+1)}(S - |\langle\boldsymbol{S}\rangle|)(S+1+|\langle\boldsymbol{S}\rangle|). \quad (14)$$

At lower temperatures, the long range order is well established and the fluctuation or excitation from the ordered state can be described by magnon excitation. A momentum loss by the scattering by a magnon is

$$\int \frac{q}{\exp(Dq^2/k_{\rm B}T) - 1}q^2dq,$$

where Dq^2 is the dispersion relation of the magnons. There are various calculations on the resistivity due to this magnon scattering, an example is given as [8] (Appendix B),

$$\rho_{\rm m} = \frac{pi^3}{32}\frac{NSm\mathit{\Gamma}^2}{e^2z\hbar E_{\rm F}^3}\left(\frac{\mu_e}{m}\right)^2(k_{\rm B}T)^2. \quad (15)$$

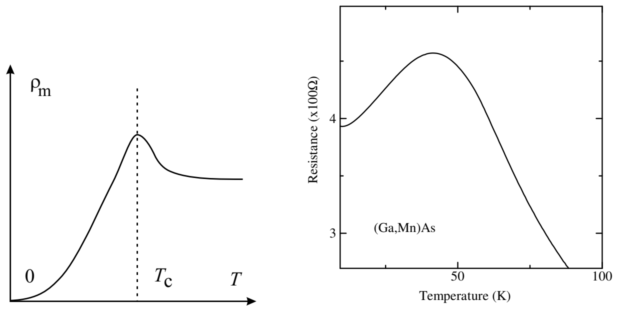

Figure 3: Left: Schematic drawing of the temperature dependence of resistivity due to spin disorder scattering. Right: Peak structure in the temperature dependence of resistance in a (Ga,Mn)As film. T_c is 45K.

This contribution is proportional to T^2.

Now we know the two extremes and the problem is what happens around T_c. It is well known that around T_c various length scales characteristic to magnetism vary rapidly and widely with temperature. Hence such a length should cross the de Broglie wavelength, which usually reults in strong back scattering. This phenomenon is called critical scattering referring to that in neutron scattering.

The treatment of critical scattering is not simple and scaling treatment has been tried, *e.g.* as [9]

$$\frac{1}{\rho}\frac{d\rho}{dT} = \frac{A_\pm}{\alpha_\pm}(|\varepsilon|^{-\alpha_\pm} - 1) + B_\pm; \quad T \to T_c^\pm; \quad \varepsilon \equiv \frac{T - T_c}{T_c}. \tag{16}$$

This results in a peak structure in the temperature dependence.

From the above discussion, the resistivity due to the spin disorder scattering can be summarized schematically as shown in the left panel of Fig.3. The right panel shows a temperature dependent resistivity of a (Ga,Mn)As film. The peak structure around T_c is common in metallic (Ga,Mn)As and was analyzed along the scenario of spin disorder scattering [10]. However different interpretations have been proposed [11] and we still have a room for research.

2.3 Spin Polaron Effect

In a system with disordered potential, the electron localization gives disorder in exchange interaction between the localized moments, results in formaton of spin (magnetic) polarons [12]. A "spin plaron" is a ferromagnetic cloud of local moments, of which interaction is mediated by a localized (but itinerant over the cloud) carrier. This description may be reversed, *i.e.* the moments are disordered and the exchange energy gives disorder and localization to a carrier. In reality these are co-operative and even in a clean system such spin polarons may be formed.

Let us roughly estimate the condition for the formation of spin polaron. The energy enhancement due to confinement into a sphere with the radius R (zero-point motion energy) is $\hbar^2\pi^2/2mR^2$. Let J_N be the energy required for a flip of a moment from the polaron ordered state and the energy of a spin polaron is given as

$$\frac{\hbar^2\pi^2}{2mR^2} + \frac{4\pi}{3}\left(\frac{R}{a}\right)^3 J_N - J_{\rm H}. \tag{17}$$

a is atomic spacing. To minimize the first two terms, the radius should be

$$R_m^5 = \hbar^2\pi a^3/4mJ_N. \tag{18}$$

Then the energy of (17) becomes

$$\frac{5\hbar^2\pi^2}{6m}\left(\frac{4mJ_N}{\pi\hbar^2 a^3}\right)^{2/5} - J_{\rm H}. \tag{19}$$

If the above is negative, spin polarons are formed.

In a clean matrix, a spin polaron is movable with dragging the cloud of polarization. Consider one atomic distance shift of a spin polaron. Let $\theta_{r,r+1}$ be the angle between neighboring spins before the shift. Then the shift requires $\theta_{r,r+1}$ of spin rotations, which means the transition matrix element is proportional to $\cos\theta_{r,r+1}$. Now we assume the spins rotate 2π in the radius R of the polaron, *i.e.*, $\theta_{r,r+1} \sim a/R$. Then the transition matrix element for whole polaron is

$$\prod_r \cos\theta_{r,r+1} \sim \left(1 - \frac{a^2}{2R^2}\right)^n \qquad \left(n = \left(\frac{R}{a}\right)^3\right). \tag{20}$$

Taking the limit of $R \to \infty$, we know that the effective mass of the polaron is at the order of $m\exp(\gamma R/a)$ (γ is of order 1).

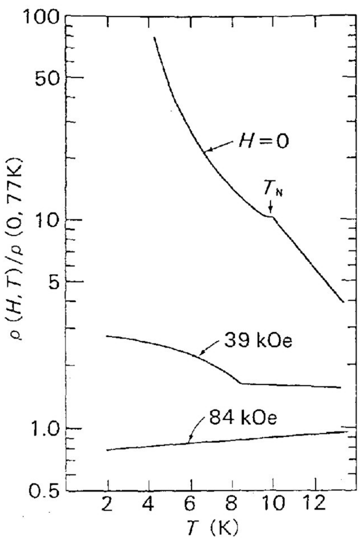

Figure 4: Temperature dependent resistivity of EuTe under three different magnetic fields (Y. Shapira et al. Phys. Rev. B **5**, 2647 (1972).).

Formation/dissolving of spin polarons usually gives drastic changes in electric transport. Here I show a representative example of EuTe. With formation of spin polarons the resistance steeply increases with decreasing temperature below the Neel point and dissolve by external magnetic field recovers metallic transport.

2.4 The RKKY Interaction

A local moment embedded in a Fermi sea causes peculiar spin polarization of conduction electrons through the exchange interaction. This gives Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between the moments and the conduction spins.

We first consider the situation that a simple charge of ze exists at the origin. According to the general theory of scattering, the asymptotic form of the scattered electron wavefunction is,

$$\Psi_l \sim Ar^{-1}\sin(kr - \frac{1}{2}l\pi + \eta_l)P_l(\cos\theta), \qquad (21)$$

where l is non-negative integer, η_l is the phase shift, P_l is the Legendre polynomial. Coefficient A is determined from the renormalization and the renormalization in the volume of a sphere with the radius R gives $2\pi A^2 R = 2l + 1$.

The change in the charge distribution due to the local moment can be obtained by putting ψ_l as the solution for no impurity ($\eta_l = 0$).

$$\begin{aligned}
&\sum_l (2l+1)\int \frac{d\Omega}{4\pi}(|\Psi_l|^2 - |\psi_l|^2) \\
&= \frac{1}{2\pi R}\sum_k \sum_l \frac{2l+1}{r^2}[\sin^2(kr - \frac{1}{2}l\pi + \eta_l) \\
&\quad - \sin^2(kr - \frac{1}{2}l\pi)] \\
&= \frac{1}{2\pi^2 r^2}\sum_l (2l+1)\int_0^{k_\mathrm{F}} \sin(2kr + \eta_l - l\pi)\sin\eta_l \, dk,
\end{aligned} \qquad (22)$$

where Friedel's sum rule

$$z = \frac{2}{\pi}\sum_l (2l+1)\eta_l(k_\mathrm{F}) \qquad (23)$$

holds. We only consider $l = 0$ term and approximate the scattering potential by a square well (within a volume Ω the potential is V_0). Then

$$\eta_0 = 2mk_\mathrm{F}\Omega_0 V_0/2\pi\hbar^2,$$

and the variation in the charge density is obtained as

$$\delta\rho = \frac{e\eta_0}{2\pi^2 r^3}\left(\cos 2k_\mathrm{F}r - \frac{1}{2k_\mathrm{F}r}\sin 2k_\mathrm{F}r\right). \qquad (24)$$

This oscillatory charge density originates from the anomaly in the sceening potential at the double of the Fermi wavelength and called Friedel oscillation.

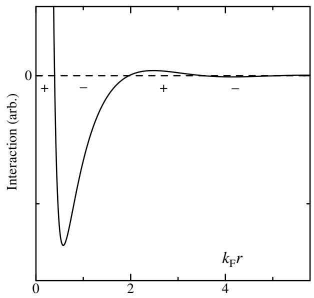

Figure 5: Example of the RKKY interaction (26).

Now we introduce a difference in the scattering potential due to the conduction electron spin such as (12). The scattered wave of electrons with up-spin $\psi_\uparrow$ and that of electrons with down-spin $\psi_\downarrow$ are different giving

$$|\psi_\uparrow|^2 - |\psi_\downarrow|^2 = 2J_\mathrm{H}\frac{m\Omega k_\mathrm{F}^2}{2\pi^3\hbar^3}\frac{1}{r^2}\left[\frac{\cos 2k_\mathrm{F}r}{2k_\mathrm{F}r} - \frac{\sin 2k_\mathrm{F}r}{(2k_\mathrm{F}r)^2}\right]. \qquad (25)$$

Hence the interaction between two moments at a distance r mediated by carriers is also oscillatory for r as

$$J_\mathrm{H}^2\frac{m\Omega k_\mathrm{F}^2}{2\pi^3\hbar^3}\frac{1}{r^2}\left[\frac{\cos 2k_\mathrm{F}r}{2k_\mathrm{F}r} - \frac{\sin 2k_\mathrm{F}r}{(2k_\mathrm{F}r)^2}\right]. \qquad (26)$$

The is the celebrated RKKY interaction. The interaction can be ferromagnetic or anti-ferromagnetic by the distance though in the average ferromagnetic. And a magnetic material due to the RKKY interaction, if it exists, is said to be ferromagnetic.

3 The Kondo Effect (in Quantum Dots)

The Kondo effect is a representative many-body effect caused by the interaction between a localized moment and conduction electrons[13]. It was discovered in diluted magnetic alloys as a "resisitance minimum" in temperature dependence. Kondo's theory opened up a way to renormalization group theory and to quantum chromo-dynamics (QCD). Hence it's impact in physics is as large as the BCS theory of superconductivity.

In the solid state field, the Kondo effect has been given attentions, *e.g.*, in the research of heavy Fermion systems[14]. And a dramatic revival was given by the discovery of the Kondo effect in quantum dot systems[15]. Because detailed study with picking up a single impurity (*i.e.*, a quantum dot) is possible, severer comparison of theories and experiments is possible, and preparation of exotic systems such as a Kondo system with a ferromagnetic lead is also possible.

Even from pedagogical point of view, a quantum dot system provides intuitive sight of the Kondo effect. Hence in this lecture I would like to introdue the Kondo effect along the quantum dot picture. It is not very difficult to understand the Kondo phenomena in bulk materials is essentially the same as that in dot systems though the outlooks are very different.

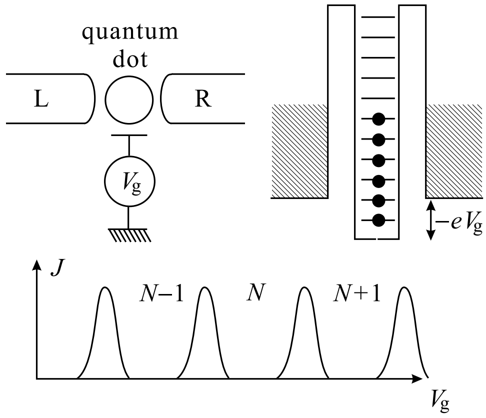

Figure 6: Upper left: "Single-electron transistor" geometry of a quantum dot. Upper right: Shematic energy diagram of a quantum dot. Lower: Schematic view of a Coulomc oscillation.

3.1 Quantum Dot

A quantum dot is a finite system isolated from the environment besides tunnel junctions. A model most frequently considered is shown in Fig.6. A dot is connected to two leads through two tunnel junctions and a gate electrode is placed to control the electrostatic potential of the dot.

The effect of quantum confinement is taken into account by a discrete energy spectrum ϵ_j and for the Coulomb interaction between electrons we take "constant (U) interaction approximation". Then the total energy of the dot with N-electrons is

$$E_N = \sum_{j=1}^{N} \epsilon_j + {}_N C_2 U. \tag{27}$$

The electrochemical (thus the electrostatic potential is included) potential of the N-th electron is

$$\mu_N = \epsilon_N + (N-1)U \tag{28}$$

The discreteness in μ_N results in discrete source-drain current peaks against the gate voltage, which is called Coulomb oscillation. Finite tunneling probability from the electrodes gives life-time width Γ. The life time τ of the N-th level is given from the tunneling Hamiltonian H_T as

$$\begin{aligned}\frac{1}{\tau} &= \sum_{\alpha=L,R;k} \frac{2\pi}{k} |\langle \alpha k | H_T | j \rangle|^2 \delta(\epsilon_k - \mu_N) \\ &= \frac{2\pi}{\hbar} \nu (|\gamma_L|^2 + |\gamma_R|^2),\end{aligned} \tag{29}$$

where $\gamma_{L,R}$ are tunnel-couplings to leads and ν is the density of states in the leads.

3.2 Electric Conductance

To calculate electric conductance through a quantum dot, let us adopt the impurity Anderson model,

$$H = H_{\rm leads} + H_{\rm dot} + H_{\rm T} \tag{30}$$

$$H_{\rm dot} = \sum_\sigma \epsilon_0 d_\sigma^\dagger d_\sigma + U_\uparrow^\dagger d_\uparrow d_\downarrow^\dagger d_\downarrow, \tag{31}$$

$$H_{\rm leads} = \sum_{\alpha=L,R} \sum_{k\sigma} \epsilon_k c_{\alpha,k\sigma}^\dagger c_{\alpha,k\sigma}, \tag{32}$$

$$H_{\rm T} = \sum_{\alpha=L,R} \sum_{k\sigma} (\gamma_\alpha c_{\alpha,k\sigma}^\dagger d_\sigma + {\rm h.c.}). \tag{33}$$

Let us consider the case $U = 0$, where the essence can be seen easier. $H_{\rm T}$ is treated as a pertubation. Tunneling probability from wavenumber k' in lead L to k in lead R can be written as

$$\begin{aligned}\Gamma_{L\to R} = 2\sum_k \sum_{k'} \frac{2\pi}{\hbar} \left| \langle Rk' | \hat{T} | Lk \rangle \right|^2 \\ \times \delta(\epsilon_{Rk'} - \epsilon_{Lk}) f(\epsilon_{Lk} - \mu_L)[1 - f(\epsilon_{Rk'} - \mu_R)],\end{aligned} \tag{34}$$

where f is the Fermi distribution function and $\hat{T}$ is a transition matrix (T-matrix) recursively defined as

$$\hat{T} = H_{\rm T} + H_{\rm T} \frac{1}{\epsilon - H_0 + i\delta} \hat{T}. \tag{35}$$

Here $H_0 \equiv H_{\rm leads} + H_{\rm dot}$. If we simply approximate $\hat{T}$ with $H_{\rm T}$, (34) becomes Fermi's golden rule.
T matrix can be calculated by *e.g.*, Green function $\hat{G}$ (Appendix C) as

$$\begin{aligned}\langle Rk' | \hat{T} | Lk \rangle = \langle Rk' | (H_{\rm T} + H_{\rm T} \hat{G} H_{\rm T} | Lk \rangle \\ = V_R^* \langle d | \hat{G} | d \rangle V_L, \\ \langle l | \hat{G} | d \rangle = \frac{1}{\epsilon - \epsilon_0 + i\Gamma}.\end{aligned}$$

From the expressions of current $I = e(\Gamma_{L\to R} - \Gamma_{R\to L}$ and voltage $V = (\mu_L - \mu_R)/e$, the conductance G is obtained as

$$G = \frac{dI}{dV} = \frac{2e^2}{h} \int d\epsilon \frac{2\Gamma_L \Gamma_R}{(\epsilon - \epsilon_0)^2 + \Gamma^2} \left(-\frac{\partial f(\epsilon)}{\partial \epsilon} \right). \tag{36}$$

Here $\Gamma = \Gamma_L + \Gamma_R$ and $\Gamma_\alpha = \pi\nu|\gamma_\alpha|^2$, and

$$-\frac{\partial f(\epsilon)}{\partial \epsilon} = \frac{1}{4k_{\rm B}T} \frac{1}{\cosh^2[(\epsilon - \mu)/2k_{\rm B}T]} \tag{37}$$

is a well known function with the peak at μ with the width $k_{\rm B}T$ and can be replaced with a delta function $\delta(\epsilon - \mu)$ when $k_{\rm B}T \ll \Gamma$ resulting in a Breit-Wigner type formula

$$G = \frac{2e^2}{h} \frac{4\Gamma_L \Gamma_R}{(\mu - \epsilon_0)^2 + \Gamma^2}. \tag{38}$$

When $k_{\rm B}T \gg \Gamma$, the Lorentz function in (36) is replaced by a delta function, resulting in

$$G = \frac{2e^2}{h} \frac{\Gamma_L \Gamma_R}{\Gamma} \frac{1}{2k_{\rm B}T} \frac{1}{\cosh^2[(\mu - \epsilon_0)/2k_{\rm B}T]}. \tag{39}$$

3.3 s-d Hamiltonian

The transport we have calculated in the previous sub-section is so called "sequential tunneling", which is an incoherent combination of two independent tunneling. In this sub-section, we consider coherent "co-tunneling", which occurs through a virtual state in a dot and is dominant in Coulomb blocked regions.

As shown in Fig.6, the number of electrons is fixed between peaks (Coulomb peaks) of a Coulomb oscillation (Coulomb valleys), and in a constant interaction model the spin of the ground state S is 1/2 for an odd N and 0 for an even N. For a while we consider the region $N = 1$ for simplicity.

Since $\mu_1 = \epsilon_0$, $\mu_2 = \epsilon_0 + U$, the energies needed to add an electron to the dot (E^+) and to extract an electron (E^-) are

$$E^+ = \mu_2 - \mu = \epsilon_0 + U - \mu \quad \text{and}$$
$$E^- = \mu - \mu_1 = \mu - \epsilon_0.$$

The condition for the Coulomb blockade is $E^\pm \gg \Gamma, k_B T$.

In the blocked region, the virtual state of an extra electron has an extra energy of E^+, thus has a finite life time of $\hbar/E^+$. Therefore if an extra electron can tunnel into another lead within the life time, transport through this virtual state is possible, which is called "co-tunneling".

To derive an effective Hamiltonian to treat spin-flip process through the dot, we first erase the freedoms in the lead from (33). We introdue the unitary transformation

$$\begin{cases} c_{k\sigma} = (\gamma_L^* c_{L,k\sigma} + \gamma_R^* c_{R,k\sigma})/\gamma \\ \bar{c}_{k\sigma} = (-\gamma_R c_{L,k\sigma} + \gamma_L c_{R,k\sigma})/\gamma \end{cases}, \tag{40}$$

where $\gamma^2 \equiv \gamma_L^2 + \gamma_R^2$. It is easy to see $\bar{c}_{k\sigma}$ does not couple to the dot and we ignore it. Next we take the second order pertubation of H_T or applying the Schrieffer-Wolff transformation to get

$$\begin{aligned} H_\mathrm{eff} = &\sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} \\ &+ J \sum_{kk'} \Big[\hat{S}_+ c_{k'\downarrow}^\dagger c_{k\uparrow} + \hat{S}_- c_{k'\uparrow}^\dagger c_{k\downarrow} \\ &+ \hat{S}_z (c_{k'\uparrow}^\dagger c_{k\uparrow} - c_{k'\downarrow}^\dagger c_{k\downarrow}) \Big]. \end{aligned} \tag{41}$$

Here $\hat{S}$ is the spin operator of the dot and the coupling constant $J = \gamma^2(1/E^+ + 1/E^-)$. It is easy to see the interaction term in (41) is in the form of (12).

The s-d Hamiltonian means an anti-ferromagnetic coupling between the local moment S and the conduction electrons in the leads. The ground state of the whole system is a many-body spin-singlet (S=0) state called "Kondo singlet". The binding energy of a Kondo singlet corresponds to a Kondo temperature T_K given as

$$k_B T_\mathrm{K} = D e^{-J\nu} = \sqrt{E^+ E^-} e^{-J\nu}, \tag{42}$$

where D is the band width in the leads and given in the second equation for the present condition[16]. For $T \ll T_\mathrm{K}$, the Kondo singlet completely screens the local moment and the conduction electron pass through the singlet freely with the conductance of the maximum value of (38), *i.e.* $(2e^2/h)4\Gamma_L\Gamma_R/(\Gamma_L+\Gamma_R)^2$. This is called "unitary limit" and can be viewed as a resonant tunneling with a width of $k_B T_\mathrm{K}$. What is peculiar is that the resonance occurs just at the Fermi level irrespective of the position of the dot electrochemical potential.

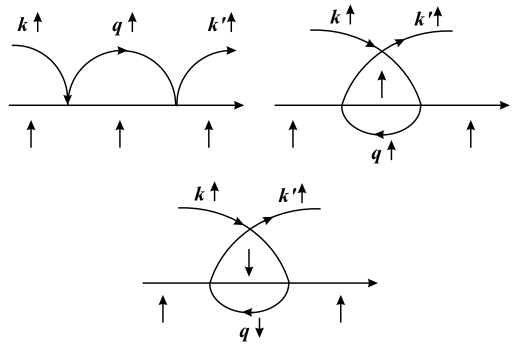

Figure 7: T-matrix diagrams for the second order of H_J. The horizontal lines indicate spin states in the dot. Right-going curves indicate electrons and left-going ones holes.

3.4 Conductance of a Kondo State

Weak Coulping Region ($T \gg T_\mathrm{K}$)

We write the last term proportional to J in (41) as H_J. In this region H_J can be treated as a perturbation. We again consider T-matrix formulation (but this time we adopt perturbation) and replace H_T, H_0 in (35) with H_J and $\sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma}$ respectively. This treatment is a Born approximation of the scattering by s-d interaction. Hence in the first order perturbation, as we have already seen in Sec.2, no anomaly appears.

The first order contribution of H_J is

$$\langle \uparrow; k' \uparrow | \hat{T}^{(1)} | \uparrow; k \uparrow \rangle = J/2 \tag{43}$$

There are three processes in the second order contribution $\langle \uparrow; k' \uparrow | \hat{T}^{(2)} | \uparrow; k \uparrow \rangle$ as shown in Fig.7. The two processes shown in the upper panel are without spin-flip while the one in the lower contains spin-flip. The first one only contains electron propagators while the others contain a hole propagator in the intermediate state. Summation of the first two contributions is

$$\begin{aligned} &\sum_q \left(\frac{J}{2}\right)^2 \frac{1}{\epsilon - \epsilon_q + i\delta} [1 - f(\epsilon_q)] \\ &+ \sum_q \left(\frac{J}{2}\right)^2 \frac{-1}{\epsilon - (2\epsilon - \epsilon_q) + i\delta} f(\epsilon_q) \\ &= \sum_q \left(\frac{J}{2}\right)^2 \frac{1}{\epsilon - \epsilon_q + i\delta}. \end{aligned} \tag{44}$$

Here the Fermi distribution functions are simply canceled out and no anomaly appears. We assume the density of state ν in the leads is constant in the band $[-D, D]$ and the sum is calculated as

$$\left(\frac{J}{2}\right)^2 \int_{-D}^{D} d\epsilon' \nu \frac{1}{\epsilon - \epsilon' + i\delta} = \left(\frac{J}{2}\right)^2 \nu \left[\ln\left|\frac{D+\epsilon}{D-\epsilon}\right| - i\pi\right]. \quad (45)$$

The third diagram in Fig.7 contains spin flip processes and thus does not have a counter term with an electron propagator for the intermediate state due to the conservation of total spin. The contribution is calculated as

$$\begin{aligned}&\sum_q J^2 \frac{1}{\epsilon - \epsilon_q + i\delta} f(\epsilon_q) \\ =& J^2 \nu \int_{-D}^{D} \frac{1}{\epsilon - \epsilon' + i\delta} f(\epsilon') d\epsilon \\ \approx& \begin{cases} -J^2 \nu \ln|\epsilon|/D & \text{for} \quad |\epsilon| \gg k_{\rm B}T \\ -J^2 \nu \ln k_{\rm B}T/D & \text{for} \quad |\epsilon| \ll k_{\rm B}T. \end{cases}\end{aligned} \quad (46)$$

The famous log divergence in (46) is due to the Fermi-edge anomaly and appears in every order of the perturbation. Most divergent term approximation gives

$$\langle \uparrow; k' \uparrow | \hat{T} | \uparrow; k \uparrow \rangle = \frac{J/2}{1 + 2J\nu \ln k_{\rm B}T/D} = \frac{J/2}{2J\nu \ln(T/T_{\rm K})}, \quad (47)$$

which is effective for $T \gg T_{\rm K}$. This T-matrix gives the conductance as

$$G = \frac{2e^2}{h} \frac{4\Gamma_L \Gamma_R}{(\Gamma_L + \Gamma_R)^2} \frac{3\pi^2}{16} \frac{1}{\ln(T/T_{\rm K})]^2}. \quad (48)$$

Strong Coulping Region ($T \ll T_{\rm K}$)

In this region, the local moment is screened and "local Fermi liquid" is a good approximation, according to which the conductance is given as

$$G = \frac{2e^2}{h} \frac{4\Gamma_L \Gamma_R}{(\Gamma_L + \Gamma_R)^2} \left[1 - \pi^2 \left(\frac{T}{T_{\rm K}}\right)^2\right]. \quad (49)$$

From the above results and also from several numerical results, it is suggested in the whole teperature region

$$G = \frac{2e^2}{h} \frac{4\Gamma_L \Gamma_R}{(\Gamma_L + \Gamma_R)^2} \mathcal{F}\left(\frac{T}{T_{\rm K}}\right), \quad (50)$$

though this has not been rigorously shown. It should be noted that a remarkable point in the Kondo effect is that there exists a single relevant energy scale $k_{\rm B}T_{\rm K}$ and physical quantities are determined by max(T,E_Z)/$k_{\rm B}T_{\rm K}$ (E_Z is the Zeeman energy).

3.5 Mean Feild Approximation

From several approximation methods of the Kondo singlet, I here introduce a pseudofermion mean field theory[17], which gives an intuitive view of the Kondo resonance.

In the s-d model (41), we introduce pseudofermion creation operator $f_\sigma^\dagger$, which creates a spin state $|\sigma\rangle$ in the dot. Under the condition

$$f_\uparrow^\dagger f_\uparrow + f_\downarrow^\dagger f_\downarrow = 1, \quad (51)$$

we can rewrite spin operators and Hamiltonian as

$$\hat{S}_+ = f_\uparrow^\dagger f_\downarrow, \quad \hat{S}_- = f_\downarrow^\dagger f_\uparrow, \quad (52)$$

$$\hat{S}_z = (f_\uparrow^\dagger f_\uparrow - f_\downarrow^\dagger f_\downarrow)/2, \quad (53)$$

$$\begin{aligned} H =& \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} \\ &+ \sum_\sigma E_\sigma f_\sigma^\dagger f_\sigma + J \sum_{kk'} \sum_{\sigma\sigma'} f_\sigma^\dagger f_{\sigma'} c_{k'\sigma'}^\dagger c_{k\sigma}, \end{aligned} \quad (54)$$

where $E_{\uparrow,\downarrow} = \epsilon_0 \pm E_Z$.

Now the spin-singlet coupled "field" of quantum dot spin $|\sigma\rangle = f_\sigma^\dagger|0\rangle$ and Fermi's sea $\prod c_{k\sigma}^\dagger|0\rangle$ is expressed by the mean field $\langle f_\sigma^\dagger c_{k\sigma}\rangle$ and the fluctuation around it is ignored, *i.e.*,

$$f_\sigma^\dagger f_{\sigma'} c_{k'\sigma'}^\dagger c_{k\sigma} \to \langle f_\sigma^\dagger c_{k\sigma}\rangle f_{\sigma'} c_{k'\sigma'}^\dagger + \langle f_{\sigma'} c_{k'\sigma'}^\dagger\rangle f_\sigma^\dagger c_{k\sigma}.$$

By using an "order parameter" of the spin singlet

$$\langle \Xi \rangle = \frac{1}{\sqrt{2}} \sum_k (\langle f_\uparrow^\dagger c_{k\uparrow}\rangle + \langle f_\downarrow^\dagger c_{k\downarrow}\rangle), \quad (55)$$

the Hamiltonian is approximated as

$$\begin{aligned} H_{\rm MF} =& \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_\sigma E_\sigma f_\sigma^\dagger f_\sigma \\ &- \sum_{k,\sigma} (\sqrt{2}J\langle\Xi\rangle c_{k\sigma}^\dagger f_\sigma + \text{h.c.}) + 2J|\langle\Xi\rangle|^2 \\ &+ \lambda \left(\sum_\sigma f_\sigma^\dagger f_\sigma - 1\right). \end{aligned} \quad (56)$$

The last term is added to perform Lagrange's undetermined multiplier method to take (51) into account.

In the absence of the Zeeman term ($E_\uparrow = E_\downarrow = 0$), $H_{\rm MF}$ can be viewed as to express a resonant tunneling through an "energy level" of $\tilde{E}_0 = \epsilon_0 + \lambda$, and with a "tunnel coupling" of $\tilde{\gamma} = -\sqrt{2}J\langle\Xi\rangle$. Hence the center of the Kondo resonance is $\tilde{E}_0$ and the width is $\tilde{\Lambda}_0 = \pi\nu|\tilde{V}|^2$. And the condition (51) meansj $\tilde{E}_0 = \mu$. $\tilde{\Lambda}_0$ is calcluated as

$$\tilde{\Lambda}_0 = \pi\nu \left|\sqrt{2}J\langle\Xi\rangle\right|^2 = D\exp[-1/2\nu J], \quad (57)$$

which is in accordance with (42).

4 TMR and GMR

The most important application of spin dependent transport at present is tunneling magnetoresistance (TMR) and giant magnetoresistance (GMR) devices.

4.1 Interface Scattering

GMR comes from the magnetization inversion of layers in a magnetic superlattice and is explained by the difference in the lifetimes of up and down spin electrons, in other words, spin-dependent scattering potentials.

Here we consider an A-B magnetic superlattice, in which the potential for electrons v is different for materials (A, B: α) and spins (+, $-$: s). According to ref.[18], in a single-band Hubbard model with the Hartree-Fock approximation,

$$v_{\alpha s} = v_\alpha^0 - \frac{1}{2} s U_\alpha m_\alpha, \tag{58}$$

where v_α^0 is the potential for paramagnetic state, U_α is the on-site Coulomb repulsion and m_α the magnetic moment per site. At the interfaces the spin dependence of the potential difference $v_{As} - vBs$ works as the spin-dependent scattering potential.

Now we adopt simple two current model, that is, we assume the spin relaxation time is much longer than the life time of carriers so that we can treat the currents of electrons with up and down spins independently.

$$j = j_\uparrow + j_\downarrow = (\sigma_\uparrow + \sigma_\downarrow) E. \tag{59}$$

In the ferromagnetic configuration, one can write the resistance of an electron parallel (anti-parallel) to the magnetization from an interface as ρ_+ (ρ_-). Hence

$$\rho_{\uparrow F} = N\rho_+, \quad \rho_{\downarrow F} = N\rho_-, \tag{60}$$

for a lattice with N interfaces. The total resistance is written as

$$\rho_F = \frac{N\rho_+\rho_-}{\rho_+ + \rho_-}. \tag{61}$$

Next in the anti-ferromagnetic configuration, half of the layers are parallel and the others are anti-parallel for electrons both with up and down spins. Therefore

$$\rho_{\uparrow AF} = \rho_{\downarrow AF} = \frac{N}{2}\rho_+ + \frac{N}{2}\rho_-, \tag{62}$$

giving

$$\rho_{AF} = \frac{N(\rho_+ + \rho_-)}{4}. \tag{63}$$

And the magnetoresistance ratio is given as

$$\mathrm{MR} = \frac{\rho_{AF} + \rho_F}{\rho_F} = \frac{(\rho_+ - \rho_-)^2}{4\rho_+\rho_-}. \tag{64}$$

If we introduce the resistance ρ_0 due to the disorder in addition to the magnetic disorder so far considered, the ratio is given as

$$\mathrm{MR} = \frac{(\rho_+ - \rho_-)^2}{4(\rho_+ + \rho_0)(\rho_- + \rho_0)}, \tag{65}$$

simply indicates the disorder in the specimen reduces the ratio.

4.2 Spin Relaxation

The breakdown of the above two current model is brought about by spin relaxation. At present four spin relaxation processes have been found to be important in solids:

- Elliot-Yafet (EY)
- D'yakonov-Perel' (DP)
- Bir-Aronov-Pikus (BAP)
- hyperfine interaction (HF).

Non-equilibrium distribution of spins caused by interfaces or spin-injections are brought back into equlibriu by these mechanisms, which can be obstacles for spintronics. Usually supressions of these effects are important issues in research and development though, sometimes, the relaxation would help fast device action.

Elliot-Yafet

It is easy to guess that if there exists the spin-orbit interaction,

$$H_{\mathrm{so}} = \frac{\hbar}{4m^2} (\nabla v_s \times \hat{\boldsymbol{p}}) \cdot \hat{\boldsymbol{\sigma}}, \tag{66}$$

where v_s is spin-independent potential, an ordinal momentum scattering by phonons, impurities, etc. can cause spin relaxation [19] because the momentum $\hbar\boldsymbol{k}$ nor the spin σ_z is no longer a good quantum number. In other words, the eigenstates should be written in the forms

$$\psi_{k\uparrow}(\boldsymbol{r}) = [a_k(\boldsymbol{r})|\uparrow\rangle + b_k(\boldsymbol{r})|\downarrow\rangle] e^{ikr}, \tag{67}$$

$$\psi_{k\downarrow}(\boldsymbol{r}) = [a_{-k}^*(\boldsymbol{r})|\downarrow\rangle - b_{-k}^*(\boldsymbol{r})|\uparrow\rangle] e^{ikr}. \tag{68}$$

In usual conditions the spin orbit interaction can be treated as a perturbation and the hybridization $|b|$ is estimated as $\langle H_{\mathrm{so}}\rangle/\mathit{\Delta} E$, where $\mathit{\Delta} E$ is the energy distance between the band states with the same momentum.

In the case of simple single valley direct semiconductors (like a part of III-V semiconductors), the spin scattering time τ_s due to the EY mechanism is obtained as

$$\frac{1}{\tau_s(E_k)} \sim \left(\frac{\mathit{\Delta}_{\mathrm{so}}}{E_g + \mathit{\Delta}_{\mathrm{so}}}\right)^2 \left(\frac{E_k}{E_g}\right)^2 \frac{1}{\tau(E_k)}, \tag{69}$$

where $\tau(E_k)$ is the momentum relaxation time.

D'yakonov-Perel'

The two wavefunctions in (68) are transformed to each other by time reversal and space inversion operation. Hence, if the space inversion symmetry is broken as in compound semiconductors, another mechanism of spin relaxation exists [20]. According to Kramer's theorem $E_{k\uparrow} = E_{-k\downarrow}$, and the inversion asymmetry causes $E_k \neq E_{-k}$. Therefore

$$E_{k\uparrow} \neq E_{k\downarrow}, \tag{70}$$

which means there is an "effective intrinsic magnetic field" $\boldsymbol{B}_i(\boldsymbol{k})$ in the crystalline. Then the interaction term can be written as

$$H_{\rm DP}(\boldsymbol{k}) = \frac{1}{2}\frac{e\hbar}{m}\boldsymbol{\sigma}\cdot\boldsymbol{B}_i(\boldsymbol{k}). \qquad (71)$$

This term causes spin presession with the traverse of electrons and being combined with momentum scattering, causes spin dephasing.

Bir-Aronov-Pikus

This spin relaxation mechanism occurs through electron – hole exchange interaction [21], which is expressed by a point contact type interaction

$$H_{\rm BAP} = A\hat{\boldsymbol{S}}\cdot\hat{\boldsymbol{J}}\delta(\boldsymbol{r}). \qquad (72)$$

Here $\hat{\boldsymbol{S}}$ is the electron spin operator, $\hat{\boldsymbol{J}}$ is the total angular momentum operator for holes. This mechanism is effective for the spin relaxation of electrons in p-type semiconductors.

Hyperfine Interaction

This relaxation is caused by the point contact interaction of electron spins with nuclear spins and well known to cause relaxation in nuclear spins, *e.g.*, in nuclear magnetic resonance (NMR) experiment. The effective Hamiltonian is written as

$$H_{\rm HF} = \frac{8\pi}{3}\frac{\mu_0}{4\pi}g_0\mu_{\rm B}\sum_i \hbar\gamma_i\hat{\boldsymbol{S}}\cdot\hat{\boldsymbol{I}}_i\delta(\boldsymbol{r}-\boldsymbol{R}_i). \qquad (73)$$

The HF mechanism is recently proven to be important in the spin relaxation in quantum dot systems[22].

4.3 Tunneling Mangetoresistance

This time we consider an effective tunnel Hamiltonian for electrode to electrode tunneling. We replace the operator d in (33) with a and adopt Fermi's golden rule (*i.e.*, T-matrix is simply proportional to H_T) obtaining

$$H_T = \sum_{kk'\sigma}\hat{T}^{\sigma}_{k'k}a^{\dagger}_{k'\sigma}a_{k\sigma} + \text{h.c.}. \qquad (74)$$

Here to take into account the ferromagnetism we put a superscript of spin σ to the T-matrix. Then the transition probability (34) becomes

$$\Gamma^{\sigma}_{L\to R}(V) = \frac{2\pi}{\hbar}\sum_{kk'\sigma}|\hat{T}^{\sigma}_{k'k}|^2 f(\epsilon_{k\sigma})[1-f(\epsilon_{k'\sigma})]\delta(\epsilon_{k\sigma}-\epsilon_{k'\sigma}+eV), \qquad (75)$$

where we have introduced the source-drain bias V.

The current for σ-spin is then written as

$$I_\sigma(V) = e\left[\Gamma^{\sigma}_{L\to R}(V) - \Gamma^{\sigma}_{R\to L}(V)\right]. \qquad (76)$$

And substitution of (75) gives

$$I_\sigma(V) = \frac{2\pi e}{\hbar}\langle|\hat{T}^{\sigma}_{k'k}|^2\rangle \times\int_{-\infty}^{\infty}\mathcal{D}_{L\sigma}(\epsilon-eV)\mathcal{D}_{R\sigma}(\epsilon)[f(\epsilon-eV)-f(\epsilon)]d\epsilon. \qquad (77)$$

If we adopt simple constant density of states model, the conductance is given as

$$G_\sigma(V) \approx \frac{2\pi e^2}{\hbar}\langle|\hat{T}^{\sigma}_{k'k}|^2\rangle\mathcal{D}_{L\sigma}\mathcal{D}_{R\sigma}. \qquad (78)$$

Thus for "ferromagnetic" (parallel) configuration of magnetization, the conductance is

$$G_F = G_{F\uparrow} + G_{F\downarrow} \propto \mathcal{D}_{ML}\mathcal{D}_{MR} + \mathcal{D}_{mL}\mathcal{D}_{mR}, \qquad (79)$$

where M and m denote majority and minority spins. Similarly for anti-ferromagnetic configuration

$$G_{AF} = G_{AF\uparrow} + G_{AF\downarrow} \propto \mathcal{D}_{ML}\mathcal{D}_{mR} + \mathcal{D}_{mL}\mathcal{D}_{MR}. \qquad (80)$$

Therefore the tunneling magnetoresistance ratio is simply given as

$$\text{TMR} = \frac{R_{AF}-R_F}{R_F} = \frac{G_F-G_A}{G_A} = \frac{2P_LP_R}{1-P_LP_R}, \qquad (81)$$

where the spin polarization P_α is defined as

$$P_\alpha = \frac{\mathcal{D}_{M\alpha}-\mathcal{D}_{m\alpha}}{\mathcal{D}_{M\alpha}+\mathcal{D}_{m\alpha}} \qquad (82)$$

Recently observed giant TMR [23] cannot be explained within this framework because it utilized spin selectivity of barriers though the extention to include the effect is not difficult.

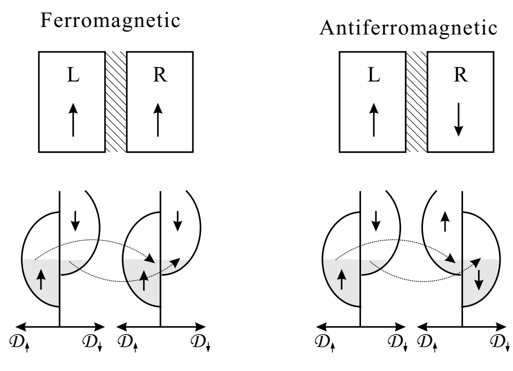

Figure 8: Tunneling magnetoresistance device. Left: Ferromagnetic configuration. Right: Anti-Ferromagnetic configuration. Lower panels show schematic diagrams of density of states for minority and majority spins.

5 Manipulation of Spins in Transport

As we have seen in the previous section, the spin orbit interaction causes a presssion of spin with the spatial traverse of an electron, which can act as a source of dephasing in combination with momentum scattering. However in ballistic transport, in which momentum scattering can be ignored, the spin-orbit interaction can be utilized to control the direction of the spins. In this section, we will briefly review the research from this view point.

5.1 Spin-Orbit Interaction

Here we ignore momentum scattering, then, for the spin-orbit interaction (66) can be effective for the transport when the spatial inversion symmetry is broken. As we have seen in the D'yakonov-Perel' effect, even in the bulk crystal there may be such spatial inversion asymmetry and the spin-orbit effect on transport due to such intrinsic asymmetry is called Dresselhaus effect. It is difficult to attain, however, such a ballistic transport in bulk semiconductors and we usually use a hetero-structure and the modulation doping technique. Hence, if such structure has some asymmetry, that would lead to the appearance of the spin-orbit effect. We call such an effect Rashba effect.

We restrict ourselves to so called two-dimensional electron gas (2DEG) at hetero-interfaces of III-V zinc blende type semiconductors. As most common configuration, let us take the growth direction as [001] (z-axis) and $x\|$ [100], $y \parallel$ [010]. Then the spin-orbit Hamiltonian becomes

$$H_{\rm so} = \alpha(\sigma_x k_y - \sigma_y k_x) + \beta(\sigma_x k_x - \sigma_y k_y), \qquad (83)$$

where the first term corresponds to the Rashba effect and the second one the Dresselhaus effect. The coefficient β contains square of the quantized momentum along z axis.

The Rashba spin presession can be tuned by applying a gate voltage to the plane and thus by changing the degree of asymmetry[24]. Since the proposal of spin transistor by Datta-Das[25], a number of proposals have appeared. Also utilization of the Dresselhaus term has been discussed[26, 27]. At present, however, no convincing experiment has been reported yet for the transistor action[28].

5.2 Spin Polarization without Ferromagnets

One of the present obstacles in spin transistor is the lack of high efficiency spin injection and filtering methods. Recently several ways to obtain spin current without external ferromagnetic materials have been proposed. One is of course the spin-Hall effect, for which a special session is prepared in this conference.

Another is a T-shaped quantum wire connection[29], which is predicted to be able to produce a spin current with almost 100% polarization for InAs/InGaAs heterostructures. This is a geometrical effect and requires precise control of the geometry[30].

Another utilizes a quantum point contact[31]. A combination of adiabatic selection of conductance path and the spin-orbit interaction results in spin current while no difference appears in the electric currents. These new ideas are now under experimental tests.

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Appendices

A Spin disorder scattering

Here I make a rough sketch of the derivation of the representation for the resistivity due to spin disordered scattering according to ref.[32].

We consider a kind of s-d model.

$$\mathcal{H}_{\rm int} = \sum_{i,j} G\delta(\boldsymbol{r}_i - \boldsymbol{r}_j)\boldsymbol{S}_{Ri}\cdot\boldsymbol{S}_{pj} \tag{84}$$

The scattering cross section is calculated in the first order Born approximation. Here we assume only the spin fluctuation $\boldsymbol{S}_R - \langle \boldsymbol{S}_R\rangle$ affects the resistance.

$$\begin{aligned}
&\frac{d\sigma}{d\Omega} \\
&= \left(\frac{m}{2\pi\hbar^2}\right)^2 \sum_{\alpha,\beta} w_\alpha \left| \langle\alpha| \sum_i e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i} G\boldsymbol{S}_p\cdot(\boldsymbol{S}_R - \langle\boldsymbol{S}_R\rangle)|\beta\rangle\right|^2 \qquad (85)\\
&= \left(\frac{m}{2\pi\hbar^2}\right)^2 \sum_{\alpha,\beta} w_\alpha \langle\alpha| \sum_i e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i} G\boldsymbol{S}_p\cdot(\boldsymbol{S}_{Ri} - \langle\boldsymbol{S}_{Ri}\rangle)|\beta\rangle \\
&\qquad \times \langle\beta| \sum_j e^{-i\boldsymbol{q}\cdot\boldsymbol{r}_j} G\boldsymbol{S}_p\cdot(\boldsymbol{S}_{Rj} - \langle\boldsymbol{S}_{Ri}\rangle)|\alpha\rangle \\
&= \left(\frac{m}{2\pi\hbar^2}\right)^2 \sum_{\alpha} w_\alpha \Bigg\{ \sum_{i,j} G^2 e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_i-\boldsymbol{r}_j)} \\
&\qquad \times \langle\alpha|(\boldsymbol{S}_p\cdot\boldsymbol{S}_{Ri} - \boldsymbol{S}_p\cdot\langle\boldsymbol{S}_{Ri}\rangle) \\
&\qquad\qquad \times (\boldsymbol{S}_p\cdot\boldsymbol{S}_{Rj} - \boldsymbol{S}_p\cdot\langle\boldsymbol{S}_{Rj}\rangle)|\alpha\rangle \Bigg\} \qquad (86)
\end{aligned}$$

Here $\sum_\alpha w_\alpha \langle\alpha|\cdots|\alpha\rangle$ can be calculated as

$$\frac{1}{2}S_p(S_p+1)[\langle\boldsymbol{S}_{Ri}\cdot\boldsymbol{S}_{Rj} - \langle\boldsymbol{S}_{Ri}\rangle\langle\boldsymbol{S}_{Rj}\rangle] \tag{87}$$

Now we consider a scattering by a local moment at the origin. Here we put

$$\sigma_0 \equiv \frac{1}{4\pi}\left(\frac{mG}{\hbar^2}\right)^2 S(S+1) \tag{88}$$

We define spin-correlation function as

$$\Gamma(\boldsymbol{r}_i) \equiv \frac{1}{S(S+1)})\langle\boldsymbol{S}_{Ri}\cdot\boldsymbol{S}_0\rangle - \langle\boldsymbol{S}_{Ri}\rangle\cdot\langle\boldsymbol{S}_0\rangle) \tag{89}$$

and

$$\sigma\theta \equiv \frac{d\sigma}{d\Omega} = \frac{\sigma_0}{4\pi}\sum_i \Gamma(\boldsymbol{r}_i)e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i} \tag{90}$$

Relaxation time approximation

$$\frac{\tau_0}{\tau} = \left(\frac{2\pi}{\sigma_0}\right)\int_0^\pi \sigma(\theta)(1-\cos\theta)\sin\theta d\theta \tag{91}$$

assumption: forward scattering is not effective. $\boldsymbol{r}_i = 0$

$$\rho_m = \rho_\infty\left\{1 - \frac{\langle\boldsymbol{S}\rangle^2}{S(S+1)}\right\} \quad \rho_\infty = (\text{const.})\times S(S+1) \tag{92}$$

B Magnon scattering

Consider 1-dimensional nearest neighbor Heisenberg model

$$H = -2J\sum_j \boldsymbol{S}_j\cdot\boldsymbol{S}_{j+1} \tag{93}$$

Let us define the effective field as

$$\boldsymbol{H}_j^{\text{eff}} = (-2J/g\mu_{\text{B}})(\boldsymbol{S}_{j-1} + \boldsymbol{S}_{j+1}),$$

then

$$H = -\frac{1}{2}\sum_j \boldsymbol{\mu}_j \cdot \boldsymbol{H}_j^{\text{eff}} \tag{94}$$

molecular field approximation：

$$\boldsymbol{H}^{\text{eff}} \longrightarrow \langle \boldsymbol{H}^{\text{eff}} \rangle \equiv \boldsymbol{H}$$

A classical equation of motion for spinwave.

$$\hbar\frac{d\boldsymbol{S}_j}{dt} = \boldsymbol{\mu}_j \times \boldsymbol{H} = 2J\{(\boldsymbol{S}_j \times \boldsymbol{S}_{j-1}) + (\boldsymbol{S}_j \times \boldsymbol{S}_{j+1}\} \tag{95}$$

Classical ground state : $\boldsymbol{S}_0$

$$\boldsymbol{S}_j = \boldsymbol{S}_0 + \delta\boldsymbol{S}_j \tag{96}$$

Linearized equation of motion

$$\frac{d\delta\boldsymbol{S}_j}{dt} \simeq \left(\frac{2j}{\hbar}\right)[(\delta\boldsymbol{S}_j - \delta\boldsymbol{S}_{j-1}) \times \boldsymbol{S}_0 + (\delta\boldsymbol{S}_j - \delta\boldsymbol{S}_{j+1}) \times \boldsymbol{S}_0] \tag{97}$$

$$\delta\boldsymbol{S}_j = \boldsymbol{A}_q e^{i(\omega t - qa)} \tag{98}$$

$$\begin{aligned} i\hbar\omega\boldsymbol{A}_q &= 2J(2 - e^{-iqa} - e^{iqa})\boldsymbol{A}_q \times \boldsymbol{S}_0 \\ &= 4J(1 - \cos qa)\boldsymbol{A}_q \times \boldsymbol{S}_0 \end{aligned} \tag{99}$$

approximation

$$i\boldsymbol{A}_q = \boldsymbol{A}_q \times \boldsymbol{S}_0/S \quad \rightarrow \quad \boldsymbol{A}_q \propto \boldsymbol{e}_x + i\boldsymbol{e}_y$$

In order for non-trivial (non-zero) solution of x,y components of $\boldsymbol{S}_j$

$$\begin{vmatrix} i\hbar\omega & 4JS(1 - \cos qa) \\ -4JS(1 - \cos qa) & i\hbar\omega \end{vmatrix} = 0 \tag{100}$$

Therefore the dispersion relation of magnons is given as

$$\hbar\omega = 4JS(1 - \cos qa) \tag{101}$$

$$\int \frac{q}{\exp(Dq^2/k_{\text{B}}T) - 1} q^2 dq$$

$$\rho_{\text{m}} = \frac{pi^3}{32}\frac{NSm\Gamma^2}{e^2 z\hbar E_{\text{F}}^3}\left(\frac{\mu_e}{m}\right)^2 (k_{\text{B}}T)^2 \tag{102}$$

C Calculation of T-matrix

A retarded Green's function $\hat{G}$ and non-perturbed Green's function $\hat{G}_0$ are introduced as

$$(\epsilon - H + i\delta)\hat{G} = 1, \tag{103}$$

$$(\epsilon - H_0 + i\delta)\hat{G}_0 = 1. \tag{104}$$

Since $H = H_0 + H_T$,

$$(\epsilon - H_0 + i\delta)\hat{G} = 1 + H_T\hat{G}. \tag{105}$$

Thus we can write down the Dyson equation

$$\hat{G} = \hat{G}_0 + \hat{G}_0 H_T \hat{G}. \tag{106}$$

Compared with the definition (35),

$$\hat{T} = H_T + H_T\hat{G}H_T. \tag{107}$$

We can thus calculate T-matrix from $\hat{G}$.

Let $|d\rangle$ be a quantum state in the dot. From (105), adopting the expression $\langle d|\hat{G}|d\rangle = \hat{G}_{dd}$ we obtain

$$\begin{aligned} &\langle d|(\epsilon - H_0 + i\delta)\hat{G}|d\rangle = (\epsilon - H_0 + i\delta)\hat{G}_{dd} \\ &= 1 + \sum_{\alpha k}\langle d|\gamma|\alpha k\rangle\hat{G}_{\alpha k,d} = 1 + \sum_{\alpha k}\gamma_\alpha^*\hat{G}_{\alpha k,d}. \end{aligned} \tag{108}$$

Similarly the operation $\langle\alpha k|\cdots|d\rangle$ gives

$$\begin{aligned} &(\epsilon - \epsilon_{\alpha k} + i\delta)\hat{G}_{\alpha k,d} \\ &= 0 + \langle\alpha k|\gamma|d\rangle\hat{G}_{dd} = \gamma_\alpha\hat{G}_{dd} \end{aligned} \tag{109}$$

$\hat{G}_{\alpha k,d}$ can be erased from the above two and

$$(\epsilon - \epsilon_0 + i\delta)\hat{G}_{dd} = 1 + \sum_{\alpha k}\frac{|\gamma_\alpha|^2}{\epsilon - \epsilon_{\alpha k} + i\delta}\hat{G}_{dd}$$

$$\therefore \quad \hat{G}_{dd} = \frac{1}{\epsilon - \epsilon_0 - \Sigma_d}. \tag{110}$$

Here the self-energy Σ_d is given as

$$\begin{aligned} \Sigma_d &= \sum_{\alpha k}\frac{|\gamma_\alpha|^2}{\epsilon - \epsilon_0 + i\delta} \\ &= \sum_\alpha \int d\epsilon'\nu(\epsilon')\frac{|\gamma_\alpha|^2}{\epsilon - \epsilon' + i\delta} \\ &= \sum_\alpha \int d\epsilon'\nu|\gamma_\alpha|^2\left[P\frac{1}{\epsilon - \epsilon'} - i\pi\delta(\epsilon - \epsilon')\right] \\ &= -i\Gamma. \end{aligned} \tag{111}$$

The real part only shifts the energy and is ignored.