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Temperature scaling of quantum Hall plateau transition in bilayer systems

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Abstract

We have investigated the scaling behavior of the quantum Hall plateau transition in double quantum well systems with different interlayer tunneling strengths. The scaling behavior of the localization property is found to be similar between the case when the relevant Landau level (LL) is non-degenerate and the case when two LLs associated with the two layers are accidentally degenerate. In both cases, the scaling exponent $\kappa \sim 0.4$ close to the canonical value is obtained, and it is unaffected by the in-plane magnetic field which changes the interlayer tunneling strength.

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Keywords: Bilayer quantum Hall system; Plateau transition; Localization; Scaling

1. Introduction

Two-dimensional electron system (2DES) formed at a semiconductor heterointerface, GaAs/AlGaAs in particular, offers an ideal experimental stage for various aspects of electron transport—most notably integer and fractional quantum Hall effects (QHEs) [1,2]. The QH plateau–plateau transition is one of the most thoroughly investigated examples of localization–delocalization transition [3–5]. The conventional picture of the integer QHE is that there is only one delocalized state at the middle of each disorder-broadened Landau level (LL) and the localization length ξ diverges as $|E-E_c|^{-\gamma}$, when the energy E approaches the LL center E_c. The value of the critical exponent $\gamma = 2.3$ (for spin-split LLs) is believed to be universal and only dependent on the fundamental symmetry of the system. In a sufficiently disordered sample at low magnetic field, it is possible for the disorder broadening of LLs to exceed the Zeeman energy, so that each orbital level becomes spin degenerate. In such a situation, analysis of the experimental results based on an assumption that the localization length diverges only at a single energy (like the case of spin-split LLs) has given a localization exponent $\gamma = 4.6$, that is twice as large as the spin non-degenerate case [6]. This result is intriguing in that it might suggest a different universality class for the spin-degenerate case. On the other hand, there is a different view on this situation. Some theoretical studies demonstrate that a nearly spin-degenerate LL has a pair of delocalized states, each of which has the localization property of the same universality class as the spin-split case [7–9]. So the localization length ξ diverges at two singular energies $-\Delta/2$ and $+\Delta/2$ (Δ being the energy difference between the two delocalized states and E_c is set at zero), and ξ is expressed as $\xi \sim |E-\Delta/2|^{-\gamma}|E+\Delta/2|^{-\gamma}$. In this view, the experimentally observed enhancement of the critical exponents for spin-degenerate LLs is an artifact due to the incorrect assumption that there is only one singular energy ($\xi \sim |E|^{-\gamma}$) while there are actually two. Although such a picture accounts for the apparent enhancement of the scaling exponent, it does not offer an explanation for the exact doubling of the exponent for the spin-degenerate case.

The problem on the localization properties in spin-degenerate LLs is difficult in the point that degree of

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mixing of two LLs is difficult to be controlled. Some theorists predicts that the situation similar to the spin-degenerated case may occur in bilayer quantum Hall systems where the bilayer degree of freedom plays a role analogous to spin [10,11]. Fig. 1 shows the flow diagram in the presence of tunneling energy t or spin-gap energy g. When the tunneling energy is larger than the disorder-broadening of LLs, the problem becomes rather trivial because the localization length diverges at two distinct singular energies which correspond to the delocalized states belonging to the symmetric and anti-symmetric states.

In the present work, we studied the scaling behavior of the transition peak width at the level crossing of bilayer systems which corresponds to the situation indicated by the question mark in Fig. 1. Bilayer systems have a remarkable feature that the bilayer degree of freedom is easily controllable by some methods. Its tunneling interaction can be controlled by the in-plane field and we studied the localization problems in bilayers with various tunneling interactions.

2. Experimental

The samples used in the present study were GaAs/AlGaAs double quantum wells grown by molecular beam epitaxy. The double layer structure consists of two 20 nm thick GaAs quantum wells separated by a 1.5 nm thick AlGaAs barrier layer. Silicon-doped AlGaAs layers were located on both sides of the double quantum well, offset by 40 nm thick undoped AlGaAs spacer layers. The random potential in each layer is mainly formed by the ionized donor impurities distributed in the doped layer adjacent to it, so the disorder potentials in the two layer are thought to be uncorrelated. The characteristic correlation length of the random potential is expected to be on the same order of magnitude as the spacer layer thickness, $R_{\mathrm{cor}} \approx d_{\mathrm{spacer}} = 40$ nm. This is greater than the magnetic length ℓ_B which is about 9 nm in the field range of the present study (7.5 T). Thus the present system may classified as

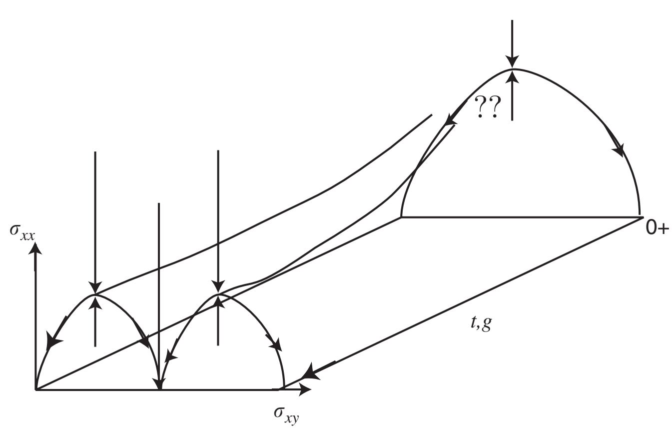

Fig. 1. The flow diagram in the presence of tunneling (adapted from Ref. [10].)

smooth potential case. The symmetric-antisymmetric tunneling gap energy Δ_{SAS} is about 1.9 meV. A Shottky front gate was used to control the carrier density and the measurements were conducted at a gate bias $V_{\mathrm{g}} = 0.0397$ V where the carrier densities of the two layers balance. At this balancing point, the electron density in each layer was $n = 2.6 \times 10^{15}\,\mathrm{m}^{-2}$ and the Hall mobility was about $\mu = 4.8\,\mathrm{m}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}$. The effective tunneling amplitude t^* can be controlled by a parallel magnetic field component. At a tilt angle θ, t^* is known to be expressed as

$$t^* = \Delta_{\mathrm{SAS}} \exp[-(d \tan\theta / 2\ell_B)^2], \quad (1)$$

where d is the layer separation.

3. Results and discussions

The experimental procedure consists essentially of measurement of the temperature dependence of the plateau transition peaks and extraction of the localization length exponent γ from the data. The width ΔB of the resistance peak is scaled with temperature T as $\Delta B \propto T^{-\kappa}$, and with the magnitude of measuring current J as $\Delta B \propto J^{-\kappa_J}$, respectively. The exponents are expressed as $\kappa = 1/\gamma z$, and $\kappa_J = 1/\gamma(z+1)$, where z is a dynamical exponent.

We first investigated the scaling behavior for the layer-non-degenerate cases. From the experimental values of the exponents κ and κ_J, we obtained values consistent with the canonical behavior, i.e. $\gamma \sim 2.3$ for spin-polarized LLs and $\gamma \sim 4.6$ for spin-degenerated LLs, and $z \sim 1$.

Next, we investigated the layer degenerate case, i.e. the scaling behavior at the crossing of LLs. Fig. 2(b) shows the magnetoresistance data under the balancing condition for different tilt angles at the base temperature (about 30 mK). Two peaks observed at $\theta = 0^\circ$ are the peaks originating from the symmetric (S0, $\downarrow$) and the antisymmetric (A0, $\downarrow$) states of the $N = 0$ down-spin LL. Upon increasing the tilt angle θ, t^* is decreased and the two peaks merge. The values of t^* calculated from Eq. (1) are given in the figure.

Fig. 3 shows the temperature scaling of the width ΔB of the resistance peak. The values of the scaling exponent κ extracted from these scaling plots are given in Fig. 4. The obtained value of the localization length exponent, $\gamma \sim 2.3$, is approximately the same as the monolayer case over the entire range of t^*, and the occurrence of $\gamma \sim 4.6$ predicted in Ref. [10] is not observed.

Ohtsuki et al. [12] have performed a numerical study of the electronic states in a double layer system with uncorrelated short-range (δ-function) disorder. Their result shows that, even for t as small as 0.1Γ, the position of delocalized states are very close to those in the case of correlated disorder. On the other hand, for the case of smooth disorder which is the case of our sample, Sørensen and MacDonald [10] have shown the possibility of existence of one localized state which has a scaling exponent twice as large as the layer non-degenerated case. The physical picture is that each time an electron tunnels to the other layer, it sees a different potential landscape so

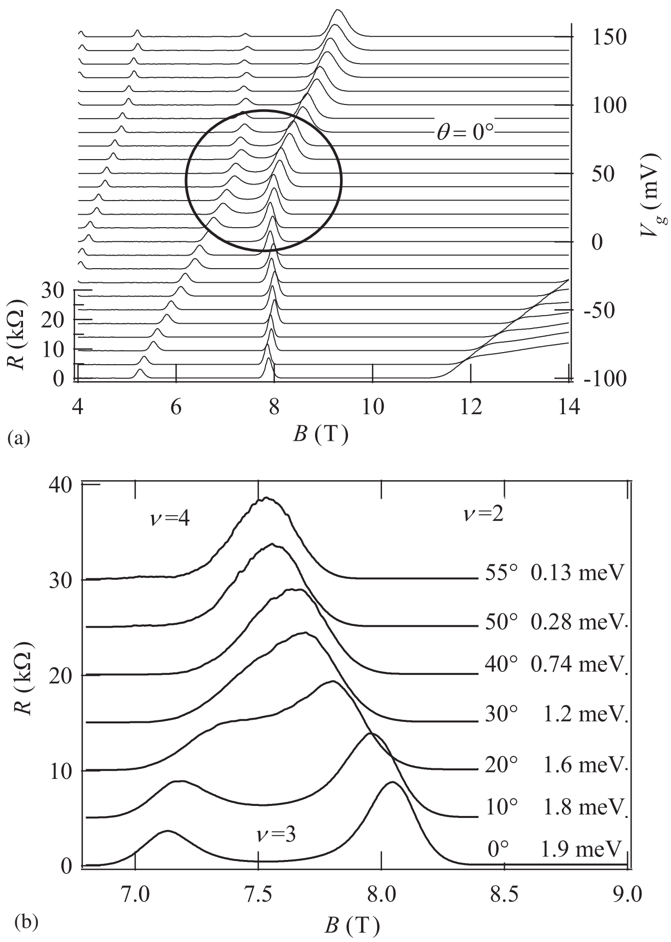

Fig. 2. (a) Magnetoresistance traces of a double quantum well sample for different values of the Shottky front gate bias. The encircled part is the region of Landau level crossing. (b) Resistance peak of quantum Hall plateau transition in a bilayer system at different tilt angles θ of the magnetic field at $V_g = 0.0397\,\text{V}$. Each trace is offset by $5\,\text{k}\Omega$. Two peaks observed at $\theta = 0^\circ$ are the peaks originating from the symmetric (S0, $\downarrow$) and antisymmetric (A0, $\downarrow$) states of the $N = 0$ down-spin LL. Upon increasing the tilt angle, the effective tunneling amplitude t^* becomes smaller and the two peaks merge.

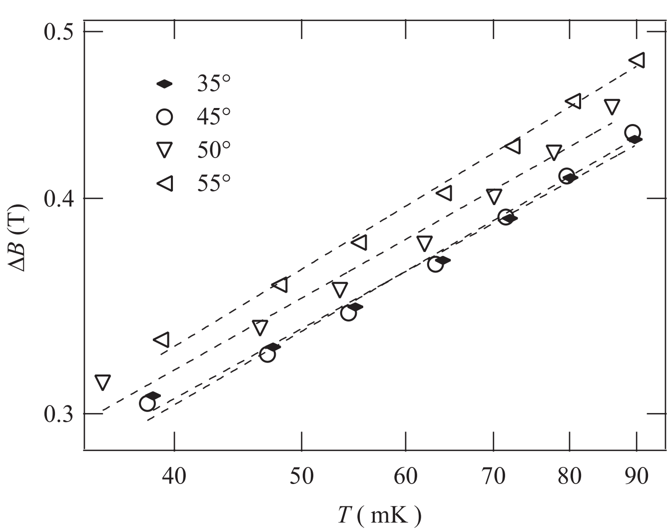

Fig. 3. Temperature dependence of the width ΔB of the resistance peak shown in Fig. 2.

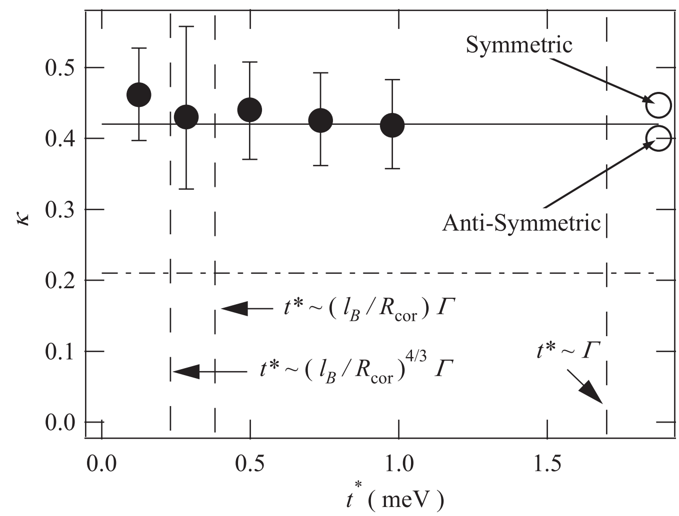

Fig. 4. The scaling exponent κ for different values of the interlayer tunneling integral t^* controlled by the tilt angle θ. The exponent $\kappa \sim 0.42$ ($\gamma = 2.3$) is approximately the same as the monolayer case over the entire range of t^*. The horizontal dot–dashed line represents $\kappa \sim 0.21$ ($\gamma = 4.6$) as observed in the spin-degenerate case, and predicted in Ref. [10] for the layer-degenerate case.

that it has a better chance of finding a different localized state when it tunnels back to the original layer. They predict that the tunneling interaction influences the percolation properties in such a way that it gives rise to doubling of the localization length for the weak tunneling interaction case $t^* < \Gamma$.

Gramada and Raikh [11] have argued that for a moderately strong interlayer coupling ($t^* > (\ell_B/R_{\text{cor}})\Gamma$), the tunneling effectively mixes the localized states in the two layers so that the picture proposed by Sørensen and MacDonald should be revised. This localized state diverges in the picture of classical percolation models [13], and the delocalization is described as percolation through cells of closed equipotentials via saddle points. This situation is similar to the original Chalker and Coddington model, so that a critical exponent that is the same as the non-degenerate case is expected in this case.

The value of Γ is estimated using the self-consistent Born approximation extended by Ando and Aoki [14] for Gaussian scatters, as $\Gamma \sim 1.7\,\text{meV}$. Using this value, the energy scales Γ, $(\ell_B/R_{\text{cor}})\Gamma$, and $(\ell_B/R_{\text{cor}})^{4/3}\Gamma$ are indicated by dashed lines in Fig. 4. In the region $(\ell_B/R_{\text{cor}})^{4/3}\Gamma < t^* < \Gamma$, the doubling of the exponent is not observed. This is consistent with the prediction of Gramada and Raikh [11]. Furthermore, even in the case of weak tunneling interaction $0.1\Gamma < t^* < (\ell_B/R_{\text{cor}})^{4/3}\Gamma$, the doubling of the exponent is not observed.

The open circles to the rightmost end of Fig. 4 represent the data for $\theta = 0^\circ$, where the two LLs of the symmetric and antisymmetric states are well separated so that they can be separately analyzed. The solid circles in the range $t^* < 1.0\,\text{meV}$ represent the cases in which the plateau transition peak can be regarded as single peak and is analyzed by assuming a single delocalization energy. In between ($1.0 < t^* < 1.5$ meV) the plateau transition peak is neither a single peak nor a well-separated double peak, so that unambiguous analysis is not possible. If we force analyze the data in this region by assuming only one delocalization energy, we tend to obtain large values of the localization length exponent, but such an analysis is clearly unphysical.

4. Conclusions

The scaling behavior of the bilayer quantum Hall system is studied. The localization length exponent γ for the layer degenerate case is found to be $\sim$2.3, that is the same as the non-degenerate case, over the whole range of tunneling interaction $0.1\Gamma < t^* < \Gamma$ investigated.

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Γ is reduced by a factor $1/\sqrt{1+(R_{\mathrm{cor}}/\ell_B)^2}$ compared to the standard self-consistent Born result for δ-function scatterers.