Experimental Verification of the Mott Relation in the Thermoelectric Effect of the Quantum Hall Systems

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Abstract. We have measured the diffusion thermoelectric power in a two-dimensional electron gas in a GaAs/AlGaAs heterostructure by employing the current heating technique at a low temperature (T=40 mK) in the magnetic field range $0 < B < 0.8$ T. We have also measured the longitudinal and the transverse resistivities and their derivatives with respect to the energy in the same sample by using a Hall bar device equipped with a metallic front gate to control the Fermi energy. We experimentally verify the Mott relation by comparing the measured diffusion thermoelectric power with those calculated by the Mott formula from the resistivities and their derivatives.

Keywords: Diffusion thermoelectric power, Quantum Hall effect, Two-dimensional electron gas, Mott formula.
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INTRODUCTION

It is well known that the thermoelectric power S and the conductivity σ of the electron systems are related to each other by the Mott formula at low temperatures ($k_BT << E_F$). Care should be taken, however, in applying the formula to the two-dimensional electron gases (2DEGs) embedded in a GaAs/AlGaAs heterostructure. The thermoelectric power contains contributions from two separate mechanisms: diffusion and phonon drag. The Mott relation is valid when only the former contribution is present. Unfortunately, however, it has been shown that the phonon drag is by far the dominant contribution in standard experiments using an external heater to introduce the temperature gradient [1]. In order to verify the Mott relation, it is therefore necessary to pick out the diffusion contribution. This can be done by employing the current heating technique that induces the gradient only in the electron temperature T_e, leaving the lattice temperature intact [2,3]. In the present study, we compare the thermoelectric power measured using the current heating technique with those calculated by the Mott formula from the resistivities and their derivatives with respect to the energy, measured in the same sample.

SAMPLE

A conventional GaAs/AlGaAs 2DEG wafer with the carrier density and the mobility n_e=4.3×10^{15} m^{-2} and μ=90 m^2/Vs, respectively, is patterned into the device geometry illustrated in Fig. 1. The device, which consists of two crossing Hall bars with metallic front gates, are designed to be much longer than the mean-free path L_{mfp}=7.3 μm of the electrons. The metallic front gate 9 enabled us to control the carrier density, hence the Fermi energy of the main Hall bar. (The metallic gate 1 is not used in the present study).

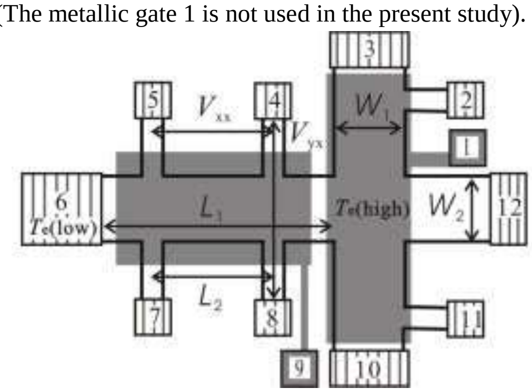

FIGURE 1. Schematic diagram of the sample. Hatched rectangles are the ohmic contacts. The main (horizontal, from 6 to 12) Hall bar (30 × 164 μm^2) contains two pairs of voltage probes (4-5, 7-8), and the distance between voltage probes is L_2=82 μm. The secondary (vertical, from 3 to 10) Hall bar (376×30 μm^2) is employed as a heater; the voltage probes (2, 11) are used for the Shubnikov-de Hass measurement to estimate T_e(high). Gray shaded rectangles are metallic front gates to control the Fermi energy.

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RESULTS

The Mott formula generalized to describe a 2DEG subjected to a perpendicular magnetic field [4] reads:

$$S_{xx} = -L_0 eT \frac{\mathrm{d}}{\mathrm{d}\varepsilon} \ln\sqrt{\sigma_{xx}^2 + \sigma_{yx}^2} \quad (1)$$

$$S_{yx} = -L_0 eT \frac{\mathrm{d}}{\mathrm{d}\varepsilon} \arctan \frac{\sigma_{yx}}{\sigma_{xx}}, \quad (2)$$

where S_{ij}, σ_{ij} are the ij components of the thermoelectric power and the conductivity tensor, respectively, and $L_0=\pi^2 k_B^2/3e^2$ is the Lorenz number. Equations (1) and (2) are rewritten in terms of the resistivity tensor ρ_{ij} and the derivative $\mathrm{d}\rho_{ij}/\mathrm{d}\varepsilon$ as

$$S_{xx} = L_0 eT \frac{\mathrm{d}}{\mathrm{d}\varepsilon} \ln\sqrt{\rho_{xx}^2 + \rho_{yx}^2} = \frac{L_0 eT}{\rho_{xx}^2 + \rho_{yx}^2}\left[\rho_{xx}\left(\frac{\mathrm{d}\rho_{xx}}{\mathrm{d}\varepsilon}\right) + \rho_{yx}\left(\frac{\mathrm{d}\rho_{yx}}{\mathrm{d}\varepsilon}\right)\right] \quad (3)$$

$$S_{yx} = L_0 eT \frac{\mathrm{d}}{\mathrm{d}\varepsilon} \arctan \frac{\rho_{yx}}{\rho_{xx}} = \frac{L_0 eT}{\rho_{xx}^2 + \rho_{yx}^2}\left[\rho_{xx}\left(\frac{\mathrm{d}\rho_{yx}}{\mathrm{d}\varepsilon}\right) - \rho_{yx}\left(\frac{\mathrm{d}\rho_{xx}}{\mathrm{d}\varepsilon}\right)\right]. \quad (4)$$

Measurements of the thermoelectric power and the resistivity are performed on the main Hall bar. The secondary Hall bar is used as a heater for the thermoelectric power measurement; it is heated by passing an ac current I_h (~30–150 nA, frequency f =13 Hz) between contancts 3 and 10. The electron temperature T_e(low) at the ohmic contact 6 is assumed to be in equilibrium with the lattice temperature, or the temperature of the mixing chamber of the dilution refrigerator. Thus the difference in the electron temperature $\Delta T_e=T_e(\mathrm{high})-T_e(\mathrm{low})$ is introduced to the main Hall bar. The longitudinal (Seebeck, S_{xx}) and the transverse (Nerrnst, S_{yx}) components of the thermoelectric power tensor are detected by probing the corresponding voltage (V_{45} and V_{48}, respectively) having the frequency $2f$ (note that $\Delta T_e \propto I^2$); then we have $S_{xx}=(V_{xx}/\Delta T_e)(L_1/L_2)$ and $S_{yx}=(V_{yx}/\Delta T_e)(L_1/W_2)$.

We can measure the longitudinal and the Hall resistance for the same area of the sample (in the absence of the temperature gradient), by passing (small) current between contacts 7 and 13. The carrier density n_e is altered by the voltage applied to the metallic front gate 9. The relation between n_e and ε is

$$dn_e = \frac{m^*}{\pi\hbar^2} d\varepsilon, \quad (5)$$

where m^* is the effective mass. Using this relation, we obtain the derivative $\mathrm{d}\rho_{ij}/\mathrm{d}\varepsilon$. The derivative is obtained either (i) by numerically differentiating the trace of ρ_{ij} vs. ε, or (ii) by employing the double lock-in technique applying ac voltage to the gate and detecting the component of ρ_{ij} that follows the frequency.

Figure 2 shows the transverse thermoelectric power S_{yx} measured with a heating current I_h=70 nA. In Fig. 2, we also plot S_{yx} calculated by Eq. (4) using the measured ρ_{ij} and $\mathrm{d}\rho_{ij}/\mathrm{d}\varepsilon$. Good agreement between the measured and the calculated traces attests to the validity of the generalized Mott relation in our sample.

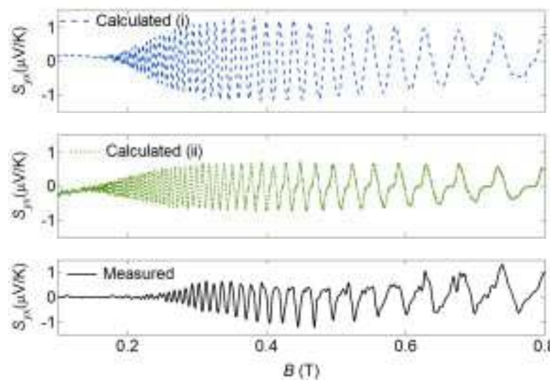

FIGURE 2. Transverse thermoelectric power S_{yx} measured directly (solid line) and that calculated by Eq. (4) using measured ρ_{ij} and $\mathrm{d}\rho_{ij}/\mathrm{d}\varepsilon$ obtained by the method (i) and (ii) described in the text (dotted lines), at T=40 mK.

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